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RESEARCH INTO TRANSPORT ASPECTS OF ICE SHEETS

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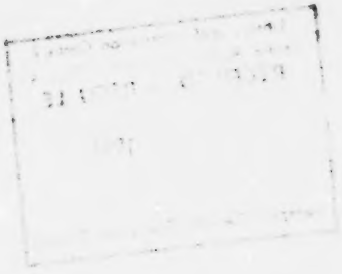

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17. Abstract

This report reviews the characteristics of floating ice sheets as a support medium for vehicle movement. Current knowledge is assessed and shortcomings are identified.

The document reviews mechanical properties of ice sheets, their static and dynamic bearing capacities, and transport operators' practices. The document summarises ice-breaking with air cushion vehicles and concludes with an identification of research requirements.

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17 Résumé

Cette étude révise les caractéristiques des plaques de glace flottantes lorsque utilisées pour le trafic véhiculaire. Les connaissances courantes sont évaluées ainsi que leurs lacunes.

Elle fait également une révision des propriétés mécaniques des plaques de glace, de leur capacité portante statique et dynamique ainsi que des pratiques courantes des transporteurs. Enfin, elle résume la technologie de la rupture des glaces par aéroglisseurs et conclut par l'identification des besoins en recherche.

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CHAPTER 1

INTRODUCTION

1.1 Background

Ice sheets are used as a support medium for vehicle movement in various situations throughout Canada. Pulp and paper companies use them in logging operations; truckers use them to move freight into isolated communities not serviced by all-weather roads; aircraft employ them as landing strips. Of increasing importance, mineral and offshore oil exploration companies are using ice sheets to move equipment to exploration sites.

Engineering practice relating to the employment, design and construction of roads on ice sheets varies widely among provincial authorities and private operators. This variation is in part due to the fact that the theory of ice sheet behaviour is not fully developed; certainly, theory is not developed to the point that it can provide a comprehensive analytical or design framework for practical problems in the field.

Whereas on the one hand an ice sheet can be used to support the movement of vehicles, it also can represent an impediment to marine navigation.

A better understanding of ice sheet behaviour is essential to the improvement of icebreaking techniques. A recent development of particular interest in this regard is the use of air cushion vehicles (ACV's) for icebreaking.

Under the sponsorship of the Transportation Development Centre, Transport Canada, research associates in the Department of Civil Engineering, University of Manitoba undertook a review and critical assessment of the current state of knowledge respecting ice sheet behaviour as it relates to transportation in Canada. This work was directed at the objectives outlined below. This report summarizes the results of this work.

1.2 Objectives of the Study

The overall objective of the project was to assess shortcomings in existing knowledge respecting the use and breakage of ice sheets for transportation, directed at identifying potentially productive areas of research. This objective was to be met by conducting the following tasks:

- a literature survey of commonly referenced material on the construction and operating practices relating to ice sheet roads, and the science of ice mechanics as it relates to ice sheets.
- a comparison of existing empirical and theoretical knowledge respecting the use of ice sheets to support vehicle movement.

- a preliminary examination of the nature, possible uses, and energy requirements of Air Cushion Vehicle icebreaking, making use of ice mechanics theory.

- interviewing engineers engaged in ice sheet road operation to clarify current engineering practices, ACV operators to gather knowledge of ACV icebreaking practice, and scientists involved in ice sheet research to determine current research areas and further research requirements.

1.3 Report Organization

The report is organized as follows:

- Chapter 2: Characteristics of Field Ice

  This Chapter outlines the physical characteristics of freshwater and sea ice of relevance to theoretical stress analysis for the material, and employment of the material as a support medium for vehicles.

- Chapter 3: Static Bearing Capacity of Ice Sheets

  This Chapter presents and compares existing theoretical approaches and engineering practices respecting the issue of the bearing capacity of ice sheets under both long-term static loading and quasi-static loading (i.e., loading under slow speed).

- Chapter 4: Dynamic Loads on a Floating Ice Sheet

  This Chapter discusses the dynamic wave effect produced by vehicles traversing ice sheets at higher speeds, and compares existing theory with experimental results and engineering practices.

- Chapter 5: ACV Icebreaking

  This Chapter examines the nature and energy requirements of Air Cushion Vehicle icebreaking.

- Chapter 6: Further Research Requirements

  This Chapter presents issues and questions warranting additional theoretical and/or experimental work to improve the safety or efficiency of ice sheet-related transporation. Research recommendations are also presented.

Two appendices are included in this report. Appendix I summarizes Olesiak's remarks on Nevel's classic paper 'Moving loads on a floating

ice sheet', suggesting possible errors in several of Nevel's formulae. Appendix II presents a discussion by Long on the response of a floating ice beam under compressive stress and a moving point load.

1.4 Background Papers

In the course of conducting the work reported here, four background papers were prepared:

- ON PROBLEMS OF THE MECHANICS OF ICE, by Z. Olesiak.

  This paper discusses the variability of the physical and mechanical properties of ice. The responses of a floating ice sheet to moving point, line and circular-distributed loads are also examined.

- MOVING LOADS ON FLOATING ICE SHEETS, by R. Long.

  This paper: describes Nevel's theory relating to the response of a floating ice sheet to a moving point load; examines fuel requirements for ACV icebreaking for a specific ACV, and assesses the effect of axial compressive stress on the critical velocity of a point load moving on a floating ice beam.

- A LITERATURE SURVEY DEALING WITH ICE SHEET ROADS, by R. Long.

  The paper summarizes and assesses widely-referenced articles concerned with the construction and operation of ice sheet roads and ice sheet experiments. Technical articles dealing with the mechanics of ice sheets are also summarized.

- REGRESSION ANALYSIS OF ICE SHEET BREAKTHROUGH DATA, by R. Long.

  This paper presents the results of a linear regression analysis performed on Gold's 1960 data [8] of moving breakthrough load and corresponding ice thickness. This data was developed from a survey of Eastern Canadian logging operators.

All of the above papers have been liberally referenced in the writing of this report. Copies of these working papers are available from the Department of Civil Engineering, University of Manitoba.

1.5 Acknowledgments

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- D. Shields and C. Man (Department of Civil Engineering, University of Manitoba), for editorial assistance.

CHAPTER 2

CHARACTERISTICS OF FIELD ICE

2.1 Introduction

Many of the problems associated with the efficient use of ice sheets as load-bearing structures arise from two important characteristics of field ice: (i) its complex structure/composition, and; (ii) its variability.

From the standpoint of structure and composition, field ice is a polycrystalline solid containing air bubbles, organic matter, and, in the case of sea ice, brine and salts as impurities. Thus, ice is usually not homogeneous. As well, the ice crystals are usually long and cylindrical and tend to show a considerable degree of orientation when found in ice covers on water. This causes the material to be anisotropic. Finally, and of great importance to the bearing capacity problem, ice is viscoelastic. Moreover, its rheological behaviour is nonlinear at stress and strain levels common in ice engineering problems.

Ice found on rivers and lakes is a highly variable material. This is because its structure, and hence its properties, is strongly affected by impurities in the water and the conditions of formation. Some conditions which have thus far been demonstrated as important are salinity, air and water temperature and temperature gradient, and water motion.

2.2 Formation and Structure

The formation and structure of ice have an important effect on its mechanical properties. Hence, an understanding of ice properties must begin with an understanding of the nucleation process.

Water exhibits two unusual properties which affect the way it freezes:

- The liquid has its maximum density at about 4°C. This property often produces an inverted temperature distribution. As water is cooled from a warm temperature, the warmer water is lighter and tends to float on top. The result is a decrease in temperature with depth. However, when some of the water is cooled below +4°C, its density is decreased and it tends to remain on the surface. As the water is cooled further, the top region in which the temperature inversion occurred (i.e., temperature increases with depth) grows. This temperature inversion is known as a thermocline. Hence, the surface water is cooler at this stage and it freezes first.
- When water reaches 0°C, it freezes and expands by 9%, making it more buoyant. Thus, under normal circumstances an ice cover forms from the top surface and grows downward. This ice cover then acts as insulation and reduces the rate at which further ice is formed.

The freezing point temperature and the temperature of maximum density are both functions of salinity - the total amount of all salts present in the water or ice. An increase in salinity reduces the thermocline effect. When the salinity increases to 24.7%, a value typical of oceans in the northern hemisphere, no thermocline occurs. In shallow waters having at least this salinity, the entire depth approaches the freezing point simultaneously. Ice forms from the top and can freeze the whole depth. A more detailed explanation of this phenomenon is given in Pounder [20] and Olesiak [19].

When the freezing water is moving quickly, the normal surface ice formation is inhibited. Instead, frazil ice, consisting of small platelets of ice distributed throughout the water depth, is formed. This frazil ice conglomerates when the water in which it is carried becomes less turbulent, as occurs when the water reaches a dam or reservoir.

Anchor ice, which forms from the bottom of a river, can be produced under certain other conditions of water motion.

The process by which ice crystals are nucleated has a pronounced effect on the final structure of ice. If ice is nucleated by dust particles on a quiet surface, the crystals will be of similar size and will tend to form with their C axis, the axis of preferred growth, vertical. This produces regular, vertically-oriented columnar grains, and an ice sheet with highly anisotropic properties.

Nucleation from fallen snowflakes causes the C axes of the resulting crystals to be randomly oriented. Crystals with a more vertical C axis orientation are favored for growth, and these eventually cut off the growth of crystals with less vertical C axis orientations.[1] A higher growth rate makes this favoring more pronounced. Again, sheets with rather anisotropic properties result.

Ice which is formed from a compressed or flooded snow cover has small crystals with completely random orientations.

---

(1) according to private communications with Dr. Sinha at NSERC labs in Ottawa

Ice formed from salt water or other impure water has dendrites reaching into the liquid phase. The impurities are rejected from the water as it freezes and collect at the base of the dendrites. From time to time these impurities are trapped as pockets in the growing crystals. These small brine or impurity pockets are usually aligned and can be seen as dotted vertical lines through the ice. The freezing rate determines the spacing and other properties of these lines. These lines are often modelled theoretically as elliptical cylinders of regular spacing.

2.3 Mechanical Properties

As mentioned, the structure of ice and hence its properties can vary a great deal depending on the conditions of its formation. This produces differences in the properties of ice when taken from different bodies of water, from different locations in the same body of water, or from different years. Even if these factors are controlled, it is difficult to determine the mechanical properties of ice for several other reasons. For one thing, its grain size is magnitudes larger than that of most engineering materials. This necessitates the use of very large test specimens if accurate measurements of macroscopic properties are to be made. In practice, many tests must be conducted on moderate sized samples, with statistical analysis being employed to establish accurate estimates of the bulk physical properties.

Many engineering materials exhibit creep and viscoelastic behaviour. Even metals exhibit pronounced creep behaviour at temperatures near

TABLE 2.1: REPRESENTATIVE STRENGTHS OF FRESHWATER ICE

Source: Pounder [20]

TYPES OF STRESS	STRENGTH (MPa)
Tension	1.5
Compression (unconfined)	3.4
Shear	0.7
Flexure (Bending)	1.7

their respective melting points. Thus there is nothing unusual in the rheological behaviour of ice. On the other hand, whether and how such behaviour should be taken into account in an engineering problem depends on the stress and strain levels, the time scale, and the temperature range defined by the problem, as well as the accuracy required. Many ice-related applied mechanics problems, including most ice pressure problems, all bearing capacity problems for moving loads, and many static bearing capacity problems, involve only elastic and primary creep behaviour.

2.3.1 Freshwater Ice

Freshwater ice contains fewer impurities than does sea ice and can better be considered as homogeneous.(1) In most cases freshwater ice can be visualized structurally as an array of long, thin vertical cylinders. Its tensile strength is greater for stresses parallel to the cylindrical axes than for those transverse to these axes. This explains why placing horizontal reinforcement such as logs or geotextiles in the ice cover should significantly increase the cover's flexural strength.

Strength values for freshwater ice are typically obtained from laboratory tests on small cylinders or small bars of rectangular cross-section with the ice at a uniform temperature. In any series of tests by the same observer, strength increases slightly as the ice temperature decreases. Representative strengths are shown in Table 2.1. Several qualifications must be made about these values. First, these are the

(1) Exceptions to this are snow ice and frazil ice.

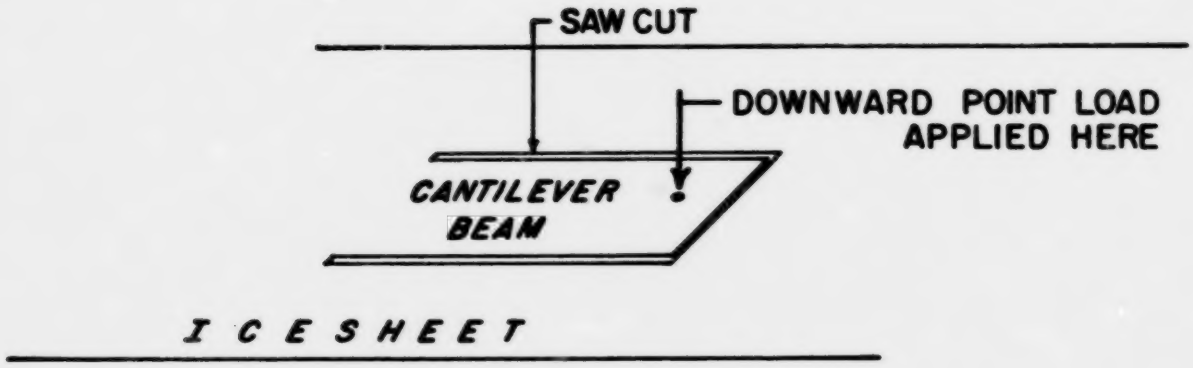

Fig. 2.1 Cantilever Beam Test

Source: Frankenstein [7]

strengths found under rapid loading for which the response is elastic (slow loading will introduce viscous effects). Second, it has been found that the sample size has a considerable effect on the results obtained. Higher strengths have been observed for small specimens, likely due to the increased probability of flaws in larger samples. Third, the results are highly variable according to the source lake (as mentioned in the previous section) and the observer.

The ultimate flexural strength of the ice is the most important strength property in applications to the load-bearing capacity problem. Because of the problems with laboratory sample tests described above, the ultimate flexural strength of ice in a floating sheet is more reliably determined from in-situ cantilever beam tests in which the temperature regime of the sheet is only slightly disturbed. See Figure 2.1 for a view of the test equipment and layout. Tests done by Frankenstein in 1959 [7] showed that the flexural strength of the ice as measured by in-situ tests was less than one-third that measured by laboratory tests on samples of the same ice (0.5 MPa vs 1.8 MPa). This gives some indication of the unreliability of laboratory values. Further discussion of in-situ experiments can be found in Olesiak [19].

2.3.2 Sea Ice

When ocean water freezes very slowly, all dissolved salts are rejected by the ice crystal lattice, and pure ice is formed. However, under normal formation conditions, the freezing rate is too rapid to allow

complete rejection, and the growing ice crystals trap a certain amount of brine. The brine freezes in fractional stages at increasingly low temperatures as different salts are rejected; the salinity of the remaining liquid thus increases as the temperature drops.

The presence of brine pockets makes sea ice a much different material than freshwater ice. While it is possible to give a single value for a mechanical property of freshwater ice that is meaningful, all mechanical properties of sea ice should be in the form of equations relating the property to the variables of temperature, salinity, and density. These three variables are conveniently combined in the variable known as brine content, ν:

$$\nu = \frac{\text{volume of fluid, brine, and air}}{\text{total ice volume}}$$

The brine content of ice decreases with decreasing temperature and decreasing water salinity. Since the fluid, brine and air pockets possess no structural strength, the strength increases as the brine content decreases. This effect is incorporated in a structural model for sea ice developed in 1958 by Assur [1]. This model assumes that the ice is comprised of parallel platelets of pure ice separated by rows of brine-containing cylinders of elliptical cross-section.

The tensile strength of sea ice is often measured by performing a ring tensile test on a hollow ice cylinder. Such tests have been performed by Graystone and Langleben [9].

Unconfined compression tests have shown the compressive strength of sea ice to vary from 1.8 to 4.8 MPa, for temperatures ranging down to -30°C [28]. The compressive strength, as is the case with the tensile strength, appears to increase with decreasing temperature and decreasing salinity, but no quantitative relation has been proposed because of the large scatter in the data. Shear tests on sea ice have shown shear strength values in the area of 1.6 MPa for ice at -6.4°C. Shear strength is also inversely related to temperature [20].

Of further concern is the fact that the strength depends on the age of the sea ice. Because sea ice over one year old is considerably less saline than annual ice due to brine drainage and summer brine replacement processes [20], it is stronger. Polar ice (permanent ice found in ice caps) is particularly strong.

A more detailed summary of sea-water ice is given by Olesiak [19].

2.4 Issues

Since the strength properties of ice in the field vary significantly from lake to lake and from year to year, it is imperative that material testing be performed continuously on an ice sheet road and the results incorporated in setting load restrictions. As mentioned earlier in this chapter, however, two important problems exist in relation to ice testing.

First, it is difficult to synthesize in a laboratory the precise temperature conditions present in ice in the field. In addition, it is impractical to test ice samples in the laboratory that are large enough to have a significant probability of containing all the structural weaknesses, such as cracks and large air pockets, present in the field ice. These difficulties require the development of reliable statistical correlations between strength properties measured in standardized laboratory tests on ice samples and the same properties measured in some standardized in-situ test. Such laboratory tests must also take into account the anisotropy of the ice, so that they measure the same property as do the related in-situ tests.

Second, there is concern that the in-situ cantilever beam test is not a reliable measure of the actual strength properties of ice in the field that are needed to allow the use of theoretical formulae for bearing capacity (see Olesiak [19]). This concern stems from two aspects of the test. First, the cut made in the ice sheet (see Fig. 2.1) allows water to penetrate upward and cover the edges of the cantilever ice beam. This changes the thermal boundary conditions and makes the observed strength values of questionable validity. Second, the ice beam is not rigidly supported at the fixed end (as is usually assumed in calculations) because the entire ice sheet is flexible. The clamping should be considered as elastic with the extent of the elastic behaviour of the fixed end being determined experimentally by measurement of the tangent to the surface at the fixed end. Recause of these drawbacks associated with the cantilever beam in-situ test as it is presently

used, it is essential that research be conducted into improving the test or developing superior tests to measure field ice strength.

CHAPTER 3

THE STATIC BEARING CAPACITY OF ICE SHEETS

3.1 Introduction

Ice sheets can be used both as travelled-ways for moving loads and as storage sites for static loads. This chapter examines the static load-carrying capability, or bearing capacity, of an ice sheet.

Static loads are loads that do not move, such as log piles placed on the ice for a winter. Quasi-static loads are loads that move at low enough velocities so that appreciable dynamic effects are not introduced to the ice sheet. From Nevel's theory of moving loads on a floating ice sheet (see Chapter 4), it can be concluded that a load is quasi-static if it moves at a velocity less than about 0.7 times the critical velocity. Critical velocity is the velocity at which deflection of the ice sheet is maximized due to dynamic effects. Depending on the ice thickness and water depth, the maximum quasi-static velocity would typically be in the neighbourhood of about 24-35 km/h (15-22 mph).

The bearing capacity problem is complicated by several factors, all having to do with the characteristics of ice in the field. The most important complicating factor is the viscoelasticity of the ice. This property means that the ice sheet really has two separate types of bearing capacity: (i) its bearing capacity for instantaneous,

quasi-static loads to which the ice responds elastically, and; (ii) its bearing capacity for static loads, which depends on the term of loading because the ice deforms continuously with time (creeps) in the vicinity of the static load.

A second complicating factor is that the lower surface of the ice plate is always at the melting temperature of about 0°C. At this temperature, the mechanical properties of the ice vary greatly with small changes of temperature. Thirdly, the mechanical properties of ice are highly dependent on conditions of formation and ambient temperature. Finally, thermal stresses may be produced in the ice due to rapid temperature changes (see Long [15]).

3.2 Theoretical Approaches

3.2.1 Theoretical Approaches for Loads of Short Duration

3.2.1.1 Introduction

For purposes of theoretical analysis, a floating ice sheet is usually assumed to respond elastically to short duration loads. Three common approaches are discussed below.

3.2.1.2 Analogy Method

The first approach, known as the analogy method, was introduced by Korunov [13]. It is based on the notion of the analogy of two ice plates which are identical except for thickness, and are assumed to be

homogeneous, isotropic, and elastic. The theory considers vertical point loads P_1 and P_2 acting on plates 1 and 2 respectively. Assuming that the bending stress distribution is linear across the plate thickness, the corresponding failure moments, M_1 and M_2, in cylindrical bending (bending about one axis) are:

$$M_1 = \sigma_f \frac{h_1^2}{6} \qquad \text{and} \qquad M_2 = \sigma_f \frac{h_2^2}{6} , \tag{3-1}$$

where: σ_f = failure stress
h_1, h_2 = thicknesses of sheets 1 and 2

Since σ_f is assumed to be the same for both sheets, it follows that:

$$\frac{M_1}{M_2} = \frac{h_1^2}{h_2^2} \tag{3-2}$$

If the bending moment M in each sheet is assumed to be proportional to the load P, (3-2) gives:

$$\frac{P_1}{P_2} = \frac{h_1^2}{h_2^2} \tag{3-3}$$

or

$$P_{all} = \frac{P_2}{h_2^2} h_1^2 , \tag{3-4}$$

where P_{all} = maximum allowable load for a cover of thickness h_1.

P_2, the maximum allowable load for a given thickness h_2, is found from an in-situ breakthrough test.

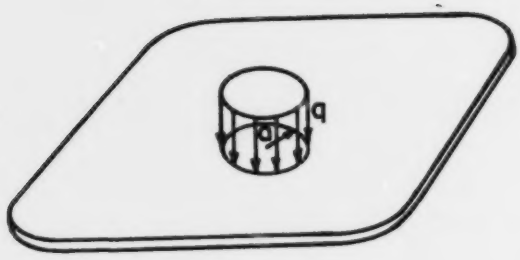

Fig. 3.1 Common Representation of Vehicle Load
Source: Kerr [12]

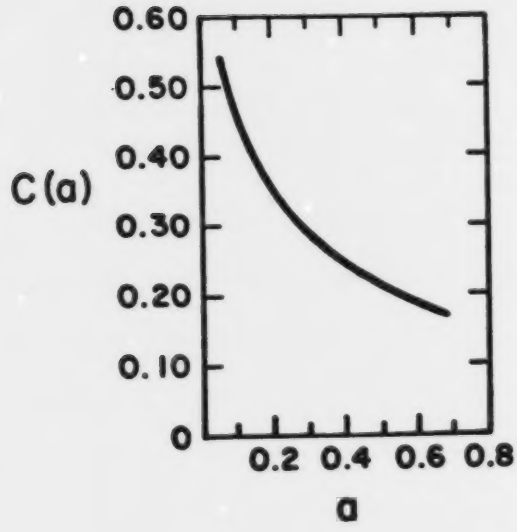

Fig. 3.2 Function Required for Use of Bernshtein's Formula
Source: Kerr [12]

The major criticism directed at this theory is that because of the temperature gradient across an ice sheet, the bending stress is not linear across the plate thickness as assumed. Advantages of the theory are that it is simple and agrees with some test results.

3.2.1.3 Method Based on Bending Theory of Elastic Plates

This method employs the classical theory of bending of thin elastic plates on elastic foundations and proceeds in three basic steps:

Step 1: Determination of σ_{max}, the maximum stress produced in the ice sheet as a function of loading.

Step 2: Determination of the load, P_{cr}, for which the first crack appears, using the criterion: $\sigma_{max} = \sigma_f$, where σ_f is the failure stress measured from an in-situ cantilever beam test as described in Chapter 2.

Step 3: Correlation of P_{cr} with the breakthrough load P_f. This must be done because according to field tests, occurrence of the first crack does not cause breakthrough.

This analysis has been done for many different load configurations, and it is usually quite complicated. For simplicity, vehicles are usually represented by a uniformly distributed load over a circular area of radius a, such that the area of the circle equals the area of the vehicle and the total load of the circle $\pi q a^2$ equals the weight of the vehicle (P). Figure 3.1 illustrates the formulation of the problem. For this loading and considering an infinite plate, Bernshtein (1929) developed the following formula for the critical load:

$$P_{cr} = \frac{1}{3(1+\nu)C(\alpha)} \sigma_f h^2 \qquad (3\text{-}5)$$

where: ν = Poisson ratio for ice
$C(\alpha)$ = a given function of $\alpha = a/L$ as shown in Fig. 3.2
a = radius of circular area

$$L = \left[\frac{Eh^3}{12(1-\nu^2)\rho}\right]^{1/4}$$

E = elastic modulus of ice
h = ice thickness
ρ = mass density of ice

The main drawback of this method is that the correlation referred to in Step 3 above is not precisely known. However, Panfilov [28] has suggested that:

$P_f = 2\ P_{cr}$, where P_f is the failure load.

3.2.1.4 Elastic Theory for Analysis of the Cracked Ice Sheet

The approach of subsection 3.2.1.3 can be carried one step further by using an elastic analysis to determine the load P_f that will not only crack the ice but will break off ice pieces near the loaded region. Various analyses of this type have been performed considering different failure mechanisms such as failure of semi-infinite and rectangular corner plates and failure of wedges. This additional analysis eliminates the empirical relationship needed in Step 3 of the previous method, but adds much complication. Details and solutions of these failure theories are given in a summary by Kerr [12].

3.2.2 Theoretical Approaches for Loads of Long Duration

The problem of the bearing capacity of an ice sheet for static loads of long duration has yet to receive a satisfactory theoretical treatment, although several attempts have been made (see Frederking [29] and Panfilov [28]). Proper treatment requires a viscoelastic plate analysis

whose form at present is uncertain because there is no consensus on the form of the constitutive equation that governs the behaviour of the ice sheet.

Investigators at Massachusetts Institute of Technology are currently investigating the viscoelastic behaviour of ice beams in an attempt to obtain more knowledge of this problem [25]. The desired output of theoretical analysis of long-term static loading on ice sheets is information on the maximum time a given load can safely be placed on a given ice sheet.

3.3 Operators' Practice

3.3.1 Gold's Formula

Despite the availability of theoretical formulae such as Bernshtein's, almost all operators of winter roads on ice in Canada use semi-empirical or empirical methods to determine the safe bearing capacity of an ice sheet for vehicles. The vehicles, of course, are usually moving and represent the quasi-static load case (unless travelling near the critical velocity, as discussed in Chapter 4).

The most common semi-empirical method employed in practice for determining safe bearing capacity is Gold's Formula, proposed in 1960 [8]. Gold's formula is:

$$P = Ah^2 \qquad (3\text{-}6)$$

where P = load in tons or tonnes
h = ice thickness in inches or cm
A = an empirical constant

TABLE 3.1 - 'A' VALUES USED IN DIFFERENT CANADIAN JURISDICTIONS

(used in $P = Ah^2$, with P in tons, h in inches)

Jurisdiction	'A' Value
Saskatchewan (Uranium City winter road)	.044
Manitoba	.041-.064
Northwest Territories	.035-.086
New Brunswick (Washademak Lake)	.022

Source: Adam [25]

This formula is used to determine the maximum permissible vehicle weight for a given ice thickness. It can be seen that the formula is similar in form to that of the analogy method, except that the 'A' factor is not determined from a test on the ice sheet in question. Instead, it is simply an empirical parameter whose value depends on observed breakthrough loading, the vehicle configuration, and the degree of safety desired.

When Gold introduced the formula in 1960, breakthrough data gathered from logging operations in Eastern Canada was used to suggest a safe 'A' value of 0.025 (imperial units). A linear regression analysis of Gold's original data performed by Long suggested that this 'A' value represents a 90% confidence level [16]. However, as shown in Table 3.1, actual values of 'A' currently used by different jurisdictions in Canada vary considerably, with most being higher than Gold's recommendation (and thus presumably less safe). The values used differ not only because different degrees of safety are accepted but because the ice itself differs in its properties depending upon geographical location. Indeed, the major drawback of Gold's formula is that it is not based on any measurement of the actual field properties of the ice sheet in question. A particular 'A' value could be safe in one region of Canada but unsafe in others. Further, the degree of safety incorporated in any particular value of 'A' could vary from year to year (as a function of ice formation conditions) and even daily (as a function of, for example, ambient temperature). On the other hand, Gold's formula is easy to use and appears to work well, assuming a judicious choice of 'A'.

TABLE 3.2. MAXIMUM PERMISSIBLE LOADS FOR VARIOUS ICE THICKNESSES

Thicknesses quoted refer to clear blue ice on lakes. The bearing capacities must be reduced by 15 percent for clear blue river ice, and by approximately 50 percent for slush ice. The values do not apply for parked loads.

Ice Thickness	Permissible Load
5 cm (2")	One person on foot
8 cm (3")	Group in single file
19 cm (7-1/2")	Passenger car (2 tonne gross)
20 cm (8")	Light Truck (2-1/4 tonne gross)
25 cm (10")	Medium Truck (3-1/4 tonne gross)
30 cm (12")	Heavy Truck (7-8 tonne gross)
38 cm (15")	9 tonnes
51 cm (20")	23 tonnes
64 cm (25")	40 tonnes
76 cm (30")	64 tonnes
91 cm (36")	100 tonnes

Source: Lumberman's Safety Association [14]

Some operators, especially loggers, do not use Gold's formula. Instead, they use an empirically-derived table of safe load-thickness combinations known as the 'logger's code', shown as Table 3.2. As well, many different rules-of-thumb are still used in different areas to determine whether the ice sheet will support a vehicle.

3.3.2 Loads of Long Duration

In practice, by far the largest use of floating ice covers to support static loads is made by the pulp and paper industry which tiers or piles pulpwood in quantity on areas of the ice to create ice landings. The pulpwood usually remains on the landing until the spring thaw when it is driven or towed downstream to the pulp or paper mill. Thus, the load is meant to eventually end up in the water, but breakthrough during winter must be guarded against to maintain the landing.

Procedures differ, but it is common among landing operators to guard against load breakthrough by building the maximum possible ice thickness through flooding the landing and by using rule-of-thumb maximum loading densities (see Rose and Silversides [21]). A common rule-of-thumb maximum density used in eastern Canada is 1125 tons per acre.

Another potentially significant use of floating ice sheets to support long-term static loads involves the build-up of floating ice islands in the high Arctic for oil and gas exploration. Such islands could be expected to carry load for periods up to 20-30 years.

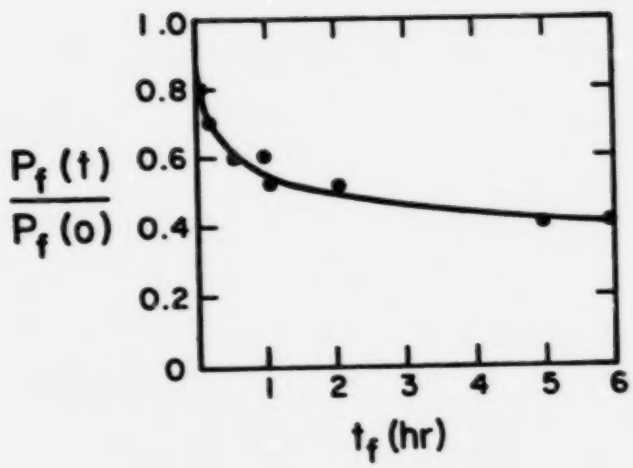

Fig. 3.3 Time Reduction of Static Breakthrough Load

Source: Kerr [12]

Because the theory for the bearing capacity with respect to long-term loads is not well developed, it could not be fruitfully applied to practice at this time. However, practice could be made more scientific by using Bernshtein's theory to determine the elastic breakthrough load, P_f, for a given ice area, and then using an experimental curve such as Figure 3.3 (which plots the ratio of long-term breakthrough load to instantaneous breakthrough load as a function of time) to take account of decreasing bearing capacity with time (due to creep).

3.4 Thermal Stresses and Cracking

The bearing capacity of an ice cover is affected by several other factors besides ice properties. For example, local water currents and local, existing cracks can weaken the ice sheet. Local currents may reduce the ice thickness in some locations on the road; a continuous program of ice thickness monitoring should be instituted to ensure that the thickness used in bearing capacity calculations is always the minimum available. Local cracks in the ice, caused by, for example, thermal stresses or ice fatigue (the failure of the ice at stresses lower than its measured strength due to many repeated loadings), will obviously reduce the bearing capacity of the ice sheet. The bearing capacity of the cracked sheet may be estimated from elastic plate theory (for short-term loads). Alternatively, bearing capacity may be estimted using the load reduction guidelines proposed by Gold for cracked ice [8].

Fig. 3.4 A Pressure Ridge on Lake Athabasca

Source: Belliveau [2]

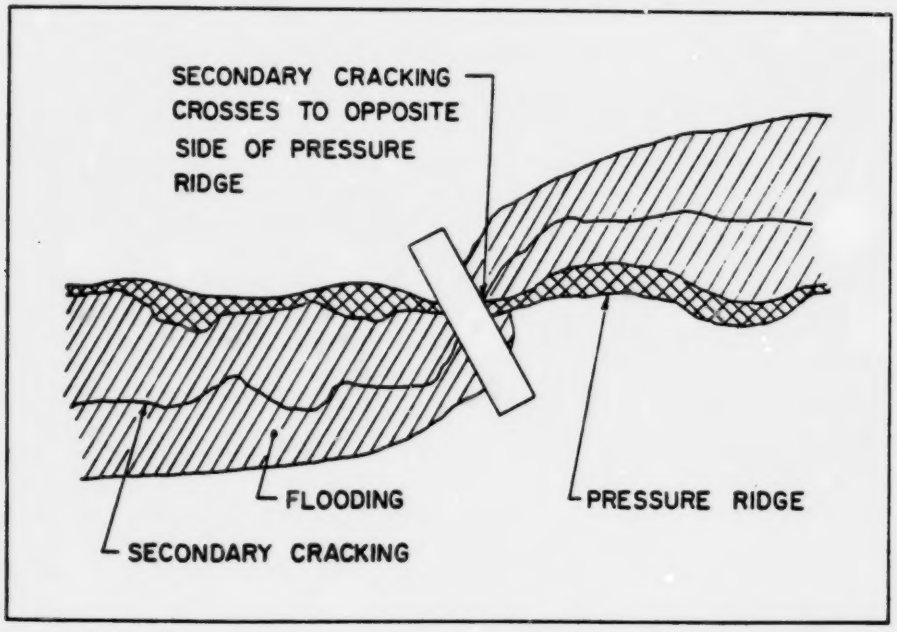

Fig. 3.5 Criterion for Locating Bridge to Cross Pressure Ridge

Source: Belliveau [2]

The problem of cracking is very pronounced when there is a sudden drop in air temperature. In an ice sheet, the temperature of the top surface is close to the air temperature, while the temperature at the bottom of the sheet is always about 0°C (for freshwater lakes). A drop in air temperature changes the temperature gradient through the ice; thermal tensile stresses are developed because contraction is induced in the ice while the sheet is partly constrained by its attachment to the shore. If stresses are sufficiently large, such as may be the case for a sudden large temperature drop, the ice sheet will crack right through. For this reason, operators in Canada usually close ice sheet roads to heavy traffic for a period of at least 24 hours following a sudden, large temperature drop.

If the air temperature remains low following the sudden drop, the large cracks through the sheet will refreeze and the road may be reopened to heavy traffic. When the temperature rises again, the new, larger volume of ice will be restrained from natural expansion by the shoreline, and buckling of the sheet may occur. This buckling takes the form of pressure ridges, which are ridges of upthrust ice. Figure 3.4 illustrates a typical pressure ridge on Lake Athabasca in northwestern Saskatchewan. On large lakes pressure ridges may rise up to 8 m in height. They usually form where large cracks existed previously, and (interestingly) often form in the same location each year. The ice in the vicinity of the pressure ridge is very weak, and must be bridged. Figure 3.5 shows a criterion for the location of the bridge which has

been used successfully by the Saskatchewan Department of Highways and Transportation on the Uranium City winter road across Lake Athabasca.

The problem of cracking and ridging appears to be a major one on large lakes used for winter transportation. On the Uranium City winter road crossing of Lake Athabasca, seven vehicle breakthroughs or partial breakthroughs have occurred in the past seven years. In each case, the ice thickness appeared to be sufficient according to Gold's Formula, but pre-existing cracks were observed in the vicinity of the breakthrough (see Belliveau [2]). In one instance, for example, a tractor fell through where two large cracks, both penetrating the full depth of the ice, intersected at a small angle to form a wedge. Hence, engineers who operate winter roads over large lakes would like to see better methods developed to monitor ice pressure buildup along the road so that the occurrence of cracks or ridges could be foreseen and the road could either be rerouted around a high stress area or loads could be reduced in anticipation of crack problems. Further, a theory which could predict which areas of the lake are most prone to the formation of large cracks and pressure ridges would be very valuable for long-term route planning.

3.5 Ice Bridges

Ice bridges are short ice crossings, most often over rivers, where the natural bearing capacity of the ice is supplemented either by flooding

to build up its thickness or by laying longitudinal reinforcement (or a combination of the two methods).

Flooding is usually accomplished by cutting a hole in the ice and pumping water onto the surface. The ice is flooded in stages so that the water from one flooding is completely frozen within a manageable period of time before another flooding is applied. The process is not continued indefinitely, because there is a limit to the thickness which can be built up, depending on climatic conditions. The ice bridge may be built by either flooding within a confined strip from shore to shore or by free flooding of the surface. In Canada, both methods are currently used, although tests by Frankenstein (1965) [7] have shown that free flooding is easier and produces a stronger bridge than does confined flooding.

For many years, ice bridges have been reinforced longitudinally. The most common form of reinforcement has been with logs, either placed individually or preconstructed into rafts. Cables have also been used. An interesting new proposal is to use geotextiles (fabric sheets) as reinforcement, especially for pressure ridge crossings. Despite this common usage of reinforcement, very few tests have been conducted to establish the actual increases in flexural strength provided by different systems. Indeed, some engineers in Canada still believe that the reinforcement weakens the bridge by causing early melting due to absorption of solar energy by the reinforcing material. However, from a consideration of the structure of ice (see Chapter 2) and from the

limited number of tests which have been performed, it appears that longitudinal reinforcement does strengthen an ice bridge if it is placed in the lower (tensile) zone of the sheet.

In practice, the bearing capacity of an ice bridge is currently determined using either Gold's formula or a rule of thumb. Theoretically, the ice bridge problem is different from the natural ice sheet problem because the plate cannot be considered of infinite extent when the crossing is short. Instead, for the case of free flooding on a narrow crossing, the theoretical model must be viewed as an infinitely long strip on an elastic foundation simply supported on both sides. Nevel carried out such an analysis in 1978 [18].

3.6 Issues

It has been stated in this chapter that the problem of the bearing capacity of an ice sheet for long-term static loads has not yet received satisfactory theoretical treatment. The problem lies mainly in a lack of knowledge of the viscoelastic (or creep) behaviour of ice sheets. Many more field experiments must be carried out to obtain curves showing the variation of static breakthrough load with time for various types of ice. Only then can empirical relationships be developed and proposed creep theories evaluated.

The chapter has discussed several theoretical approaches to the bearing capacity of ice sheets for quasi-static (short-term) loads. These

methods could be applied to actual ice sheet roads if the engineers in charge would conduct the in-situ strength and ice property tests which are required. At present, however, most engineers operating roads on ice in Canada continue to use empirical relationships such as Gold's formula to determine bearing capacity. In so doing, the actual existing strength characteristics of the given ice sheet are ignored.

With respect to quasi-static bearing capacity, one issue of interest to operators that has received very little treatment is that of the bearing capacity for a series of vehicles in close proximity. It is reasonable to expect that the method of superposition could be used with Wyman's Theory [24] of the deformations of a floating elastic plate to derive formulae for optimally safe vehicle spacings.

It has been noted that thermal stresses and the resultant cracking and ridging of the ice sheet are of major concern to engineers involved in ice sheet road use in Canada. Much research, both theoretical and practical, needs to be done in this area to improve transportation safety on the roads and improve route planning. Specific requirements are:

- development of a capability to predict the effects of thermal stresses on ice sheet bearing capacity.
- development of a capability to predict where cracks and pressure ridges are most likely to occur on a given lake or river.
- development of economical instrumentation to monitor ice pressure buildup along a road on ice.

The knowledge gained from this research will help prevent vehicle breakthrough on large frozen lakes such as Lake Athabasca where cracking is particularly evident.

CHAPTER 4

DYNAMIC LOADS ON A FLOATING ICE SHEET

4.1 The Dynamic Wave Phenomenon

When a vehicle moves over an ice sheet road at a low constant velocity, the deflection bowl created is similar to that generated by a stationary load. As speed increases, dynamic waves start to appear in the ice sheet; the moving vehicle is preceded and followed by 'humps', and the deflection under the load increases. The amplitude of these dynamic waves and the deflection under the load are maximized at a vehicle speed known as critical velocity, u_c. As speed increases above u_c, the amplitude of the waves and the deflection under the load progressively decrease.

The importance of this phenomenon lies in the fact that the increased ice deflection associated with the waves produces increased ice stress. This increased stress can cause breakage of an ice cover which would not be broken at low speeds, given the same load. On one hand, therefore, the effect reduces the bearing capacity of the ice for land vehicles; on the other hand, it increases the icebreaking potential of air cushion vehicles (ACV's).

4.2 Nevel's Theory

4.2.1 Basic Assumptions

The most widely regarded and comprehensive theoretical treatment of the problem of a moving load on a floating ice sheet is that published by Nevel in 1970 [17]. Nevel formulated the problem as that of a vertical concentrated load P (later modified to a uniform load distributed over a circular area of radius R) moving at constant velocity u on an infinite elastic plate with a Winkler foundation. The Winkler foundation is an elastic foundation such that the foundation pressure is proportional to the deflection.

To analyze the problem, Nevel made three important assumptions:

(i) the ice is homogeneous, isotropic, and elastic.

(The third assumption is reasonable where the load moves at a significant velocity; but the first and second are only rough approximations of reality).

(ii) the water is incompressible and has no viscosity.

(iii) the velocity head of the water[1] is neglected (so as to linearize Bernoulli's equation).

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[1] The velocity head of the water is its kinetic energy per unit volume, given by:

$$\text{Velocity Head} = \frac{\rho V^2}{2} \qquad (4\text{-}1)$$

where: ρ = mass density of the water
V = velocity of the water

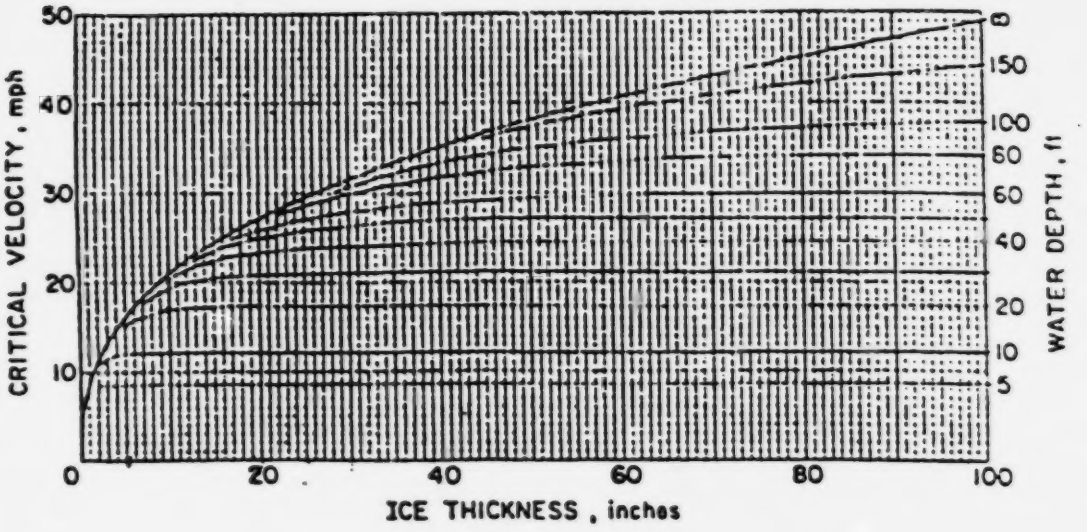

Fig. 4.1 Graph of Critical Velocity as a Function of Ice Sheet Thickness and Water Depth

Source: Nevel [17]

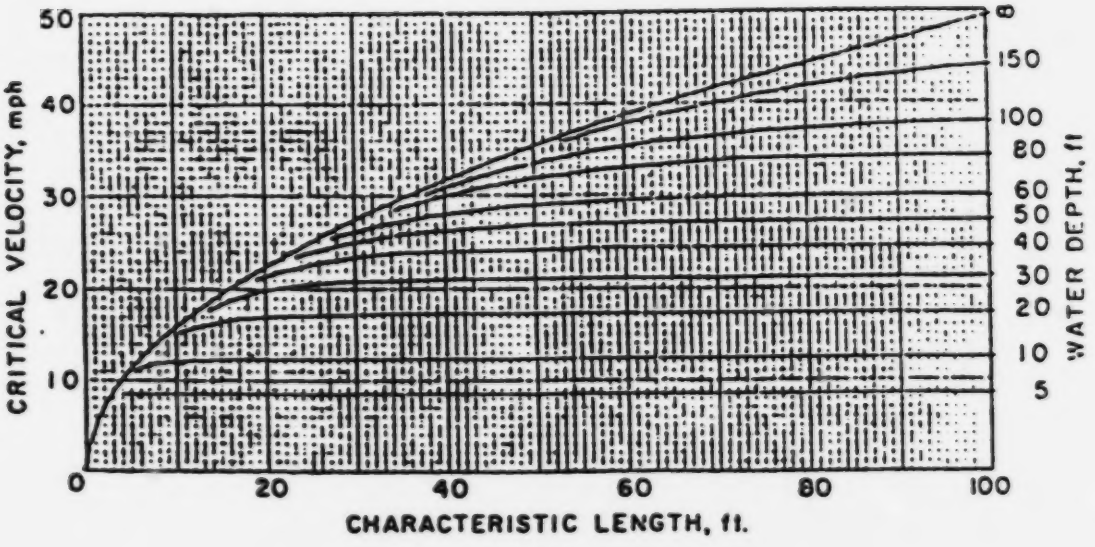

Fig. 4.2 Graph of Critical Velocity as a Function of Characteristic Length of Ice Sheet and Water Depth

Source: Nevel [17]

4.2.2. Results and Implications

Nevel derives explicit equations for the deflection of the ice, w, and the curvatures of the plate in the x and y directions, $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$ respectively.

As well, the critical velocity must be solved from two simultaneous equations, one of them transcendental. The results of the solution[2] are presented in Figures 4.1 to 4.5.

Figure 4.1 can be used to determine the critical velocity for an ice sheet, where the ice thickness (h) and water depth (H) are known.

Figure 4.2 is used to find the critical velocity as a function of H and the characteristic length[3], L, where:

$$L = \left[\frac{Eh^3}{12\,(1-\nu^2)\rho}\right]^{1/4} \qquad (4\text{-}2)$$

where: ν = Poisson ratio of ice
ρ = mass density of water

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[2] Nevel's calculations assume the following:

E = the elastic modulus of ice = 50,000 kg/cm^2 ≡ 711.4 kips/in^2
ν = Poisson ratio of ice = 0.33
mass density of ice = mass density of water = 1 gm/cm^3 ≡ 62.4 lb/ft^3

[3] The characteristic length is a parameter which represents the bending resistance of the ice sheet.

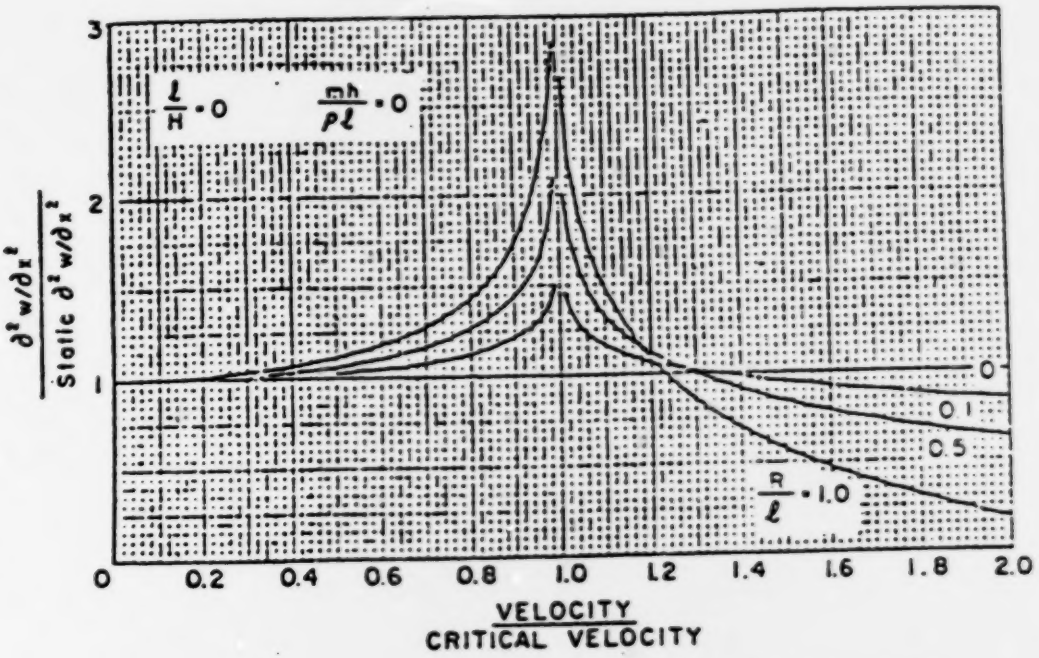


Fig. 4.3 Graph of Ratio of Curvature to Static Curvature in the x Direction as a Function of the Velocity to Critical Velocity Ratio and the Ratio of Load Radius to Ice Sheet Characteristic Length (Deep Lake Case) Source: Nevel [17]

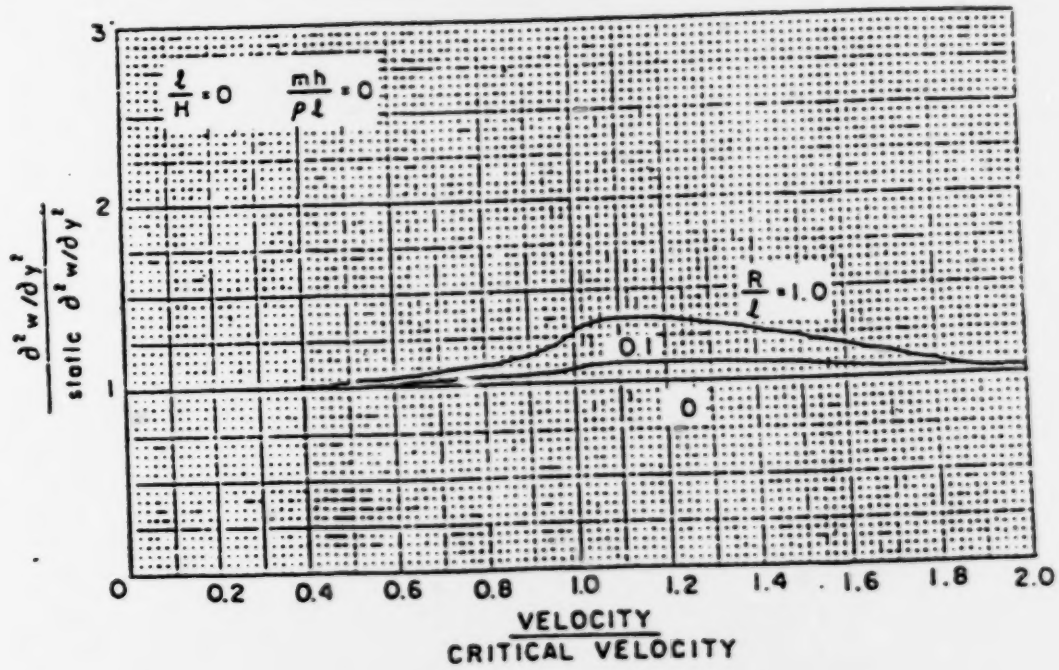


Fig. 4.4 Graph of Ratio of Curvature to Static Curvature in the y Direction as a Function of the Velocity to Critical Velocity Ratio and the Ratio of Load Radius to Ice Sheet Characteristic Length (Deep Lake Case) Source: Nevel [17]

Figures 4.3 and 4.4 show the ratios of dynamic to static curvatures in the x and y directions respectively as functions of the ratio of velocity to critical velocity (u/u_c) and the ratio of radius of loaded area to characteristic length (R/L) for the deep lake case, where

$L/H = 0$ and $\frac{mh}{\rho L} = 0$ (m is the mass density of ice). Figure 4.5 shows the deflection amplification factor (ratio of maximum dynamic deflection to maximum static deflection) as a function of the u/u_c ratio, and demonstrates Nevel's result that the deflection at critical velocity is infinite, and that the deflections at velocities greater than (about) $1.2\ u_c$ are less than the deflection under static loading.

4.2.3 Apparent Limitations to Nevel's Theory

The limitations on the predictive capability of Nevel's Theory for actual field behaviour of ice sheets, imposed by the simplifying assumptions outlined previously, have been of concern to engineers and researchers (see Shinbrot [22]). As well, further inaccuracies may arise because of mathematical error. Recent work by Olesiak [19] has shown that Nevel's Equation (36) for deflection is incorrect, and that the integrals in the subsequent formulae are also incorrect as a result. This observation indicates a need to rework Nevel's calculations. The corrections may lead to results which better agree with actual observations of the dynamic wave phenomenon in ice sheets.

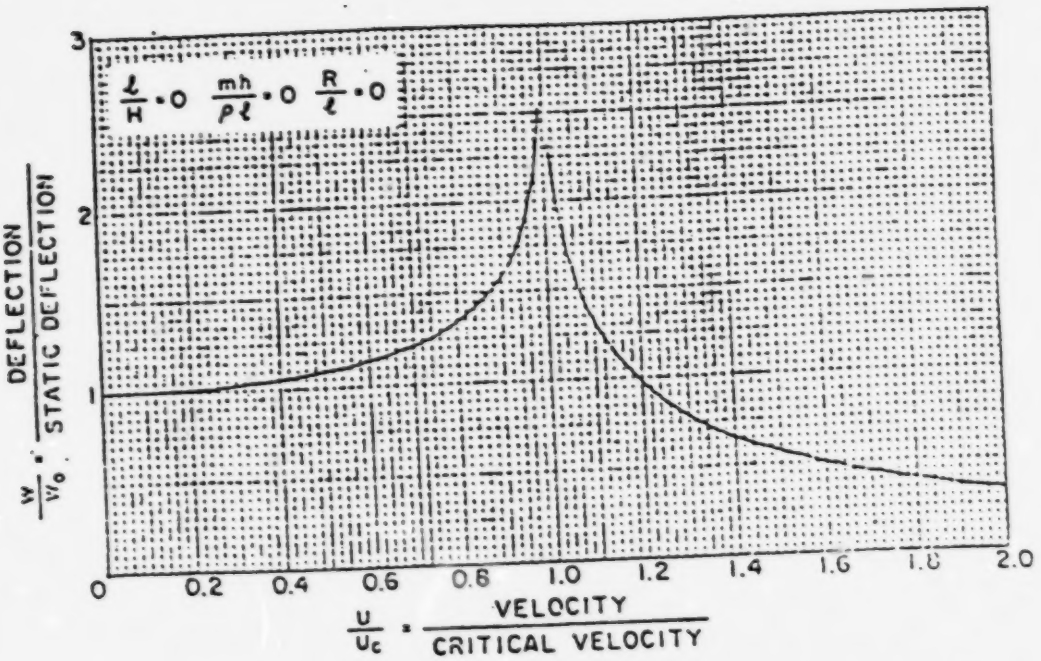


Fig. 4.5 Graph of Ratio of Deflection to Static Deflection Under Moving Load as a Function of Ratio of Load Velocity to Critical Velocity (Specific Case) Source: Nevel [17]

Source	Ice thickness (m)	Water depth (m)	Observed Critical speed (m/s)	Predicted Critical speed (m/s)	Amplification at Critical Speed for: Deflection	Total Height	Strain
Wilson (1955)	0.61	3.0	6.3	5.4	1.8	2.5	-
	0.28	3.5	5.7	5.9	1.4	2.1	-
Anderson	1.37	15.2	14.4	12.3	4.6	-	-
Eyre (1977)	0.72	36.4 (avg)	≈14.9	13.9	2.1	-	-
Beltaos Joseph Lake	0.66	1.8	4.9	4.2	2.1	2.3	-
	0.37	2.0	4.9	4.4	2.1	3.1	≈1.4
Beltaos Cold Lake Feb. 1 and 2, 1977	0.59	4.3	7.3	6.5	1.9	2.4	-
	0.65	11.6	11.0	10.4	1.9	2.5	≈1.4
	0.64	27.1	13.7	13.1	2.3	3.3	-
	0.65	54.3	15.2	13.5	2.1	2.3	≈1.4

Table 4.1 Experimental Observations of Ice Sheet Dynamic Response

Source: Beltaos [3]

4.3 Experimental Results

Only four detailed field studies on the dynamic response of ice covers have been conducted: by Wilson (1955), Anderson (1958), Eyre (1977) [6] and Beltaos (1976-77) [3]. The results of the studies are summarized in Table 4.1 (from Beltaos). It can be seen that the observed critical velocities are all within about 10% of those predicted by Nevel's theory, while tending to be higher than the predicted values. However, as would be expected, the deflection at the critical velocity is not found to be infinite as forecast by the theory. Instead, a range of 1.8 (Wilson) to 4.6 (Anderson) has been observed for the critical amplification factor (deflection amplification at critical velocity). While Anderson's high value would appear to be an anomaly, it cannot be disregarded because of the small number of measurements which have been made so far. It is possible, as Beltaos suggests, that the critical amplification factor is large but occurs only within a very narrow speed range not yet encountered in any of the field studies.

Nevel's theory predicts that the amplification of stress is generally lower than that of deflection. This was observed experimentally by Beltaos, who found the maximum strain amplification factor, occurring with the vehicle travelling near the critical velocity, to be about 1.4 as compared to 1.9 - 2.1 for the corresponding critical deflection amplification factor. Stress is proportional to strain according to the assumption that the cover responds elastically. Since it is the ice

stress and not the deflection which governs breaking, it is most important to determine the critical strain amplification factor.

One interesting and important observation was made by Eyre [6] in his study when he undertook to use a vibration transducer to measure the intensity of noise emitted during ice fracture. Eyre observed that the peak noise intensity occurred at a velocity of only about 85% of the critical velocity. If it is assumed that the intensity of noise is directly related to the ice fracture rate (number of fractures generated per unit area of ice per unit time), then the peak noise intensity velocity of .85 u_c is the most damaging velocity. This is not explained by Nevel's theory.

In general, though, the limited experimental observations have lent strong support to Nevel's theory. Nevel himself suggests that the discrepancy between the infinite critical deflection predicted by his theory and the smaller observed values may arise from three of his assumptions:

(i) Neglecting the velocity head of the water.

(ii) Neglecting the dissipation of energy in the water, but more particularly the ice.

(iii) h, H and u being assumed constant. [If these parameters are not constant in the physical situation, the problem may be transformed to a transient one (with time effects observed in the moving coordinate system) rather than a steady state one.]

These factors may also explain the discrepancy between the assymmetry of Nevel's relationship between w/w_0 (ratio of deflection at critical

TABLE 4.2 SPEED LIMITS FOR ROADS ON ICE

Source: Adam [25]

JURISDICTION	SPEED LIMIT (mph)	COMMENTS
Saskatchewan (Wollaston Lake Road)	25	Limit applies to all traffic
Saskatchewan (Uranium City Winter Road)	25	Limit reduced to 10 mph near shore. Cars usually travel at higher speeds of 30-35 mph.
Manitoba	8-12	Limit applies to loaded trucks
Ontario (Spruce Falls Power and Paper)	10	For an unreinforced ice bridge
Ontario (Kimberley-Clarke Canada Ltd.)	10	For an unreinforced ice bridge
N.W.T. (Echo Bay Road)	25	

speed to static deflection) and u/u_c, and the symmetry of the same relationship observed by Beltaos in his experiment.

4.4 Operators' Practice vs Theory

Most operators of winter roads on ice in Canada are aware of the existence of the dynamic wave effect and hence impose speed limits on the roads. Some operators impose low speeds at all times, while some require low speeds only when approaching shore due to the possibility of dynamic wave reflection and reinforcement. This shore effect has not yet been analyzed theoretically, but seems to command wide attention among both road operators and drivers.

Table 4.2 shows some speed limits imposed by different Canadian jurisdictions. According to Nevel's Theory (referencing Figure 4.1), a representative range of the critical velocity for Canadian roads would be 23-33 mph (based on an ice thickness range of 20-35 in. and a water depth in excess of 40 ft.). It would therefore appear that some operators are imposing speed limits considerably lower than necessary to avoid the critical amplification while others are allowing velocities quite close to critical velocity. In the case of the Uranium City winter road, the speed limit has been set near the critical velocity because the wave effect has not been strongly observed on the road and all vehicle breakthroughs so far have been attributed to pre-existing cracks rather than the dynamic effect [2]. It must also be remembered that if the maximum allowable load is determined by using a conservative

bearing capacity criterion (i.e. a low 'A' value in Gold's formula), the wave effect will be less significant.

The wide variation of ice sheet road speed limit practices in Canada indicates differences of opinion between engineers regarding the importance and most efficient way of avoiding the dynamic wave problem, and suggests a possible need for greater awareness among engineers of the potential use of Nevel's charts in setting efficient speed limits.

Other issues of practical concern to operators are vehicle spacing and passing and vehicle configuration. These problems are complicated, and no significant theoretical analyses have been performed. As explained, Nevel's theory considers only single point and uniformly distributed loads and a constant speed. In practice, operators have found that it is unsafe for heavy vehicles to cross the ice in close proximity to each other. Minimum allowable spacings are set ranging in the area of 900 ft. (see Adam [25]). The passing problem is often alleviated by having vehicles slow down to low speeds as they approach each other and by prohibiting the passing of a vehicle unless it is stopped. More theoretical and experimental work is required to determine the effects of vehicle configuration on the dynamic wave phenomenon.

It is also observed in Table 4.2 that several jurisdictions apply the speed limit only to loaded trucks, allowing cars or light trucks to travel at higher speeds. This is a practice consistent with the experimental observation of the finite nature of the critical amplification.

When the critical amplification is finite, a vehicle could still travel safely at the critical velocity as long as its weight is less than the maximum allowable low-speed load determined from a static bearing capacity formula divided by the critical strain amplification factor. In mathematical terms, the maximum load that could be allowed to travel through the critical velocity denoted as $P_{dynamic}$, would be (using Gold's formula for bearing capacity):

$$P_{dynamic} = \frac{Ah^2}{\text{Critical Strain Amplification Factor}} \qquad (4\text{-}3)$$

where: h = ice thickness
A = constant

Also, once the critical velocity is exceeded, according to Nevel's theory and experimental observations, the vehicle is as safe or safer from the standpoint of breakthrough as it would be at a speed which was lower than the critical velocity by the same amount. This is potentially significant to the economics of certain trucking operations across ice sheets. To this effect, certain of such operations may realize a beneficial trade-off by permitting speeds greater than the critical velocity, but with a corresponding reduction in load.

4.5 Issues

The discussion in this chapter has indicated a clear need for further research into the topic of moving loads on a floating ice sheet so that operators can more safely, efficiently, and uniformly account for the

dynamic wave phenomenon. Both theoretical (applied mechanics) and experimental work is necessary.

At the applied mechanics level, the theory of Nevel should be examined and modified in an attempt to obtain results more closely approximating reality (especially with regard to the critical amplification factor). This could be done by changing some of Nevel's assumptions as has been suggested by Nevel himself. As well, Nevel's theory should be expanded or a new theory developed to predict dynamic effects under the following common practical situations:

- an accelerating/decelerating vehicle.
- multiple vehicles at different proximities.
- a vehicle approaching shore.

At the experimental/empirical level, more tests (such as those conducted by Beltaos) must be performed on floating ice sheets to obtain a reliable value of the critical strain amplification factor which would be employed in a formula for $P_{dynamic}$. It is thought by some experimenters that the critical amplification factor may be large, but that it lies within a very narrow range of velocities. This possibility could be investigated in the laboratory using a floating ice sheet model under strict speed control unattainable in the field. Finally, further measurements of the fracture intensity (noise intensity) of the ice under a moving load should be made to check Eyre's finding that the peak fracture activity occurs at 0.85 of the critical velocity.

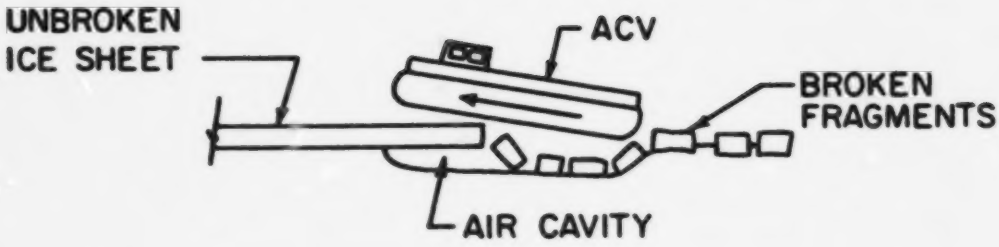
CHAPTER 5

ACV ICEBREAKING

5.1 Introduction

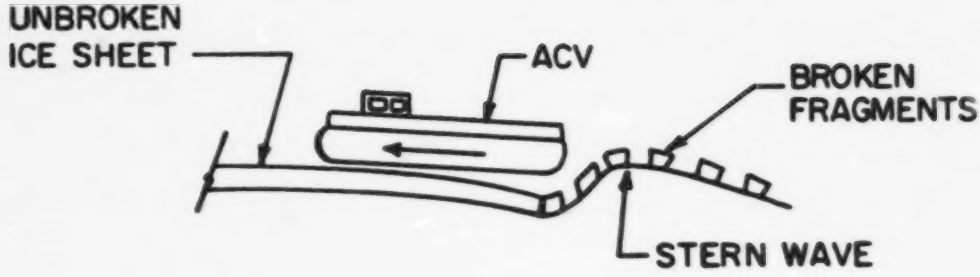
The concept of icebreaking using air cushion vehicles (ACV's) is relatively new. The potential was first discovered in the winter of 1971-72 when the ACT-100 air cushion platform travelling at about 4 mph unexpectedly broke ice up to 27 inches thick at Yellowknife. Since then, trials of the icebreaking capabilities of several other ACV's on different frozen water bodies have been carried out by agencies such as the Canadian Coast Guard [5]. Potential advantages offered by ACV icebreaking include its possible use on remote lakes that are inaccessible to conventional ice breaking ships and its ability to provide icebreaking capability to commercial ships by means of attaching air cushion platforms to the bows of these ships.

Trials have demonstrated two modes of ACV icebreaking depending upon the vehicle velocity. At low velocities, the ice is broken in the air cavity mode (see Fig.5.1a). A common, although unproven, explanation for the failure mechanism in this mode (see Dutfield [5] and Hanamoto [4]) proposes that the ACV stationed over a section of unbroken ice allows cushion air to penetrate below the ice through small fissures. This creates an air cavity under the ice. Then, according to the explanation, the resulting lack of ice sheet support coupled with the ACV bearing load breaks the ice cover over the cavity. As the ACV proceeds over the



Slow Mode

(a)



Fast Mode

(b)

Fig. 5.1 Slow and Fast Modes of ACV Icebreaking

Source: Brodland [4]

cantilever caused by the cavity, the cantilever is broken off due to cushion air pressure and the beam's own weight.

At some velocity less than the critical velocity, and in a gradual manner, the mode of ice breaking shifts to the wave action mode (see Figure 5.1b). This mode makes some use of a stern wave generated in the water under the ice behind the ACV, and also uses the dynamic waves and resulting stress amplification created in the ice sheet by the moving load. The breakage is sometimes aided by trimming down the stern of the ACV to contact the trailing wave periodically, a practice known as 'tramping'.

The object of this chapter is to examine the fuel requirement of ACV freshwater icebreaking in the wave action mode, assuming that breakage in this mode is due to dynamic ice waves only. Nevel's graph of critical velocity as a function of ice thickness (h) and the water depth (H) is used to find the fuel requirement as a function of these same two variables. As a representative ACV for the fuel study, the Bell Aerospace Voyageur 002 is considered.

5.2 The Voyageur 002

The Voyageur 002, manufactured by Bell Aerospace Canada Ltd., is an ACV owned by the Canadian Coast Guard. Its chief use at present is in breaking ice flows in the small lakes of the St. Lawrence Seaway system to prevent downstream jamming. It has also been used in ice sheet breaking trials (Parry Sound, 1974).

The dimensions of the craft are roughly 35 ft wide by 65 ft long. The vehicle can be loaded to a maximum weight of 105,000 lb., but it usually operates at 88,000 - 90,000 lbs. One disadvantage of the ACV ice-breaking concept is that the weight of the vehicle cannot be varied to a great extent. The main method used to increase weight is to build up an ice cover on the craft, but this occurs gradually. Then, the vehicle loses weight constantly in operation as the fuel load is consumed. In essence, the vehicle is restricted to a roughly constant weight, so that if it is used to crack ice of much lower thickness than its capability, energy is wasted.

5.3 Fuel Requirement Analysis

5.3.1 Assumptions

A number of assumptions have been made for purposes of analyzing the fuel requirements of ACV icebreaking. Respecting loading, the assumptions are:

(i) The tramping process simply helps to break into pieces the ice which is already cracking due to the dynamic wave (i.e. the initial breakage is not dependent on the tramping force).

(ii) The load is uniformly distributed. (This is not completely accurate if tramping is used because tramping causes the centre of gravity of the load to move backward from the centre of the craft).

As mentioned in Chapter 4, Nevel's theory predicts an infinite deflection amplification at the critical velocity, while in actual experiments this amplification factor has ranged from 1.4 to 4.6. According to the theory, strain (and hence stress) amplification is generally slightly lower than deflection amplification. So far, Beltaos (1977) is the only

investigator to have measured the critical strain amplification, and he found a value of about 1.4. It is therefore reasonable to assume that the load required to break a thickness h at the critical speed is 1/1.4 times that required at low speeds. Considering the higher values of deflection amplification factor reported by others (see Table 4.1) this value is likely conservative.

The operational fuel consumption of the Voyageur 002 is 150 gal/hr, roughly constant over its normal range of operating speeds from low speeds to 12 knots (13.8 mph). This implies that the miles per gallon increases linearly with velocity. This trend would, of course, be reversed at higher speeds until increasing speed would mean decreasing miles per gallon (as occurs for automobiles at about 28 mph). However, because a proper fuel consumption-velocity curve is not available for the Voyageur 002, it is assumed that the miles per gallon increases linearly with velocity for the full range of speeds up to the critical velocity. The gallons per mile (GPM) is then found by:

$$GPM = \frac{150}{v} \qquad (5\text{-}1)$$

where: v = velocity in mph

The assumption is only approximate, but is justified by the fact that for a reasonably large ice thickness (i.e. 30 inches), the maximum critical velocity is only 32 mph, low enough that it can be assumed that the fuel consumption rate (gal/hr) is constant.

5.3.2 Analysis for the Voyageur 002

Using the best-fit form of Gold's formula for the quasi-static bearing capacity for trucks and Bombardiers (neglecting the load distribution effect on the constant A)[4]:

$$P = 0.077\ h^2 \qquad (5\text{-}2)$$

where P = load in tons
h = ice thickness in inches
A = 0.077 taken from Long [15]

$$\text{or} \quad h = 3.60\ (P)^{1/2}\ , \qquad (5\text{-}3)$$

it is found that the maximum thickness of freshwater ice that can be broken by the Voyageur with a fixed weight of 90,000 lbs. (travelling at critical velocity) is:

$$h = 3.60\ \left(\frac{90{,}000}{2{,}000}\right)^{1/2} (1.4) = 33.8 \text{ inches.}$$

From Nevel's table, the critical velocities (u_c) corresponding to different pairs of h and H, with h varying up to its maximum of 33.8 inches, were found. The following formula was then used to determine the gallons of fuel required per mile of icebreaking.

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[4] Strictly speaking, the empirical constant in Gold's formula should be determined directly from ACV field tests. The value A = 0.077 is admittedly somewhat arbitrary. An inappropriate value for A, however, would not invalidate our method of analysis. We urge the reader to pay attention to the method rather than to the numbers henceforth in this chapter.

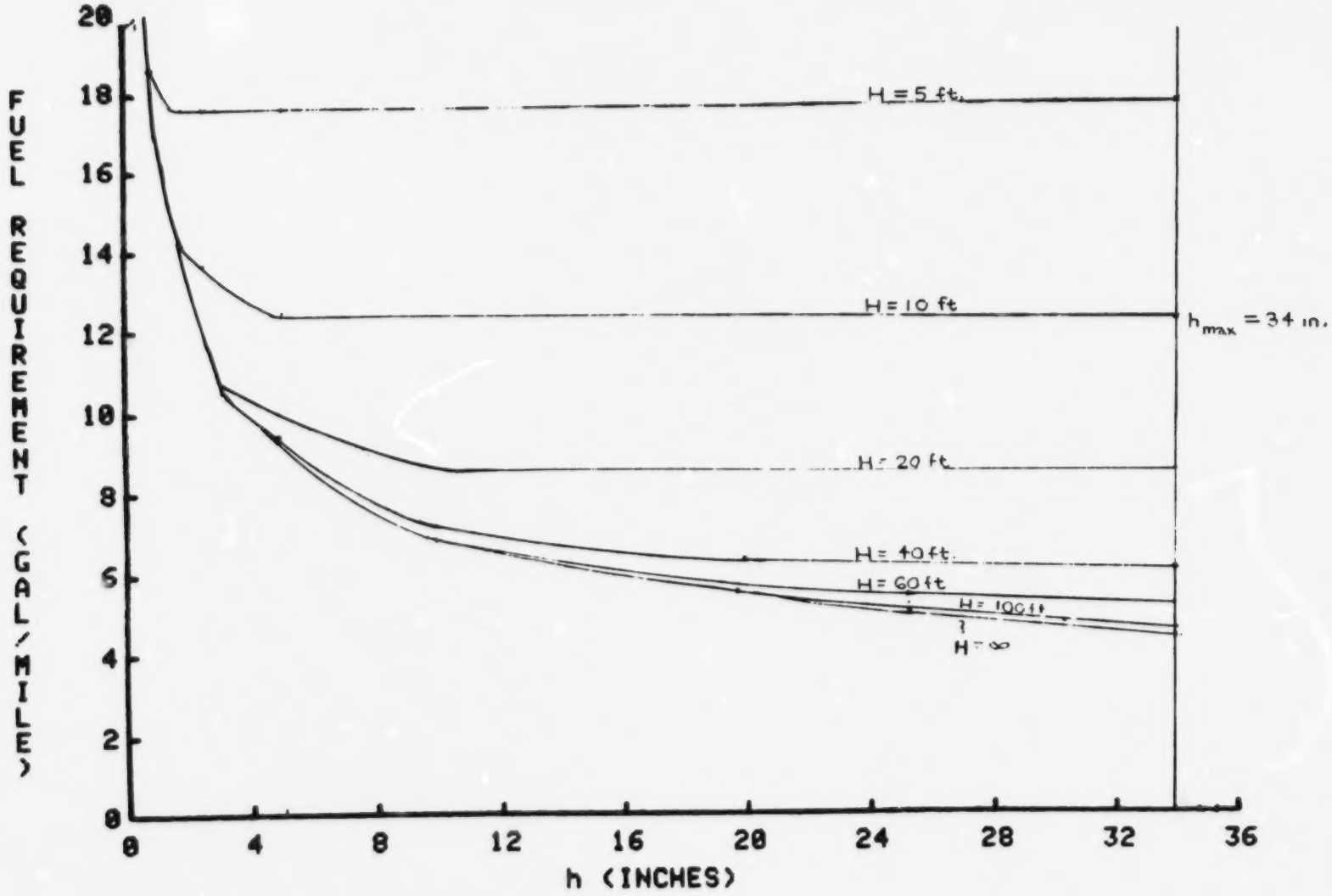

Figure 5.2: Fuel Requirement for ACV Icebreaking in Optimum Wave Action Mode Source: Long (15)

$$GPM = \frac{150}{u_c} \qquad (5\text{-}4)$$

where u_c = critical velocity

The results are shown graphically in Figure 5.2, which illustrates how the fuel requirement varies with h and H.

5.4 The Effect of Changing the Elastic Modulus

Figures 4.1 to 4.5 are based on an assumed value for E of 50,000 kg/cm^2 = 4905 MPa = 711.4 $kips/in^2$.

In Nevel's theory, E affects the critical velocity through the characteristic length, L, which is a parameter describing the ice sheet and water foundation stiffness with the units of length. Figure 4.2 is Nevel's more general graph relating the critical velocity to L and the water depth H.

The formula for the characteristic length of the ice sheet is:

$$L = \left[\frac{Eh^3}{12(1-\nu^2)\rho}\right]^{1/4} \qquad (5\text{-}5)$$

where: ρ = mass density of water

From this it is seen that as E is increased, L increases by a factor of $(\Delta E)^{1/4}$. Thus it can be seen from Figure 4.2 that at small water depths E has little effect on critical velocity, but at large depths increasing E will increase the critical velocity (for constant h) and further reduce the fuel requirement according to the assumed fuel

consumption formula. Because of the fourth root, however, a large change in E produces only a moderate change in gallons per mile.

As an example, for H = 100 ft, h = 25 inches and E = 711 kips/in^2, the GPM calculated by using L, Figure 4.2, and formula (5-5) is 5.4. If E is then increased to 1000 kips/in^2, holding the other values constant, the GPM falls to 4.7.

If a reliable value of E is available from local ice tests, it should be used in conjunction with Figure 4.2 to calculate the energy requirement.

5.5 Discussion of Results

Figure 5.2 shows that the fuel requirement for ACV icebreaking is strongly affected by the water depth. For any given ice thickness, the fuel requirement is reduced substantially as the depth increases. Thus, ACV icebreaking in the wave mode is most economical from a fuel standpoint when it is carried out over deep lakes. The fuel requirement does not vary much with the ice thickness except at large depths where the requirement is slightly greater for smaller thicknesses. Since it is just as cheap or cheaper to crack the maximum thickness as to crack a lower thickness, the optimum situation for fuel economy is to have the ACV cracking its maximum thickness (over a deep lake).

It is important to realize that there may be several operational difficulties in carrying out the optimum wave action mode of icebreaking

described above. First, to allow navigation through frozen waters, the icebreaking action must succeed in breaking the ice into sufficiently small pieces along the route. While the dynamic wave will certainly cause the ice to crack, it may not create sufficiently small pieces nor cause the pieces to separate. The tramping operation helps to effect breakup into smaller, distinct pieces, but even so, several passes of the ACV may be required to open the route sufficiently. At this time it is not known exactly how the number of passes is related to the thickness, and the requirement for additional passes would multiply the fuel cost. Second, the icebreaking could not be carried out over very rough ice or over large pressure ridges because there is a limit to the ACV's skirt height. The Voyageur 002, for example, usually operates at a skirt height of 4 ft. Third, some types of ice, such as the slush ice of the Great Lakes, are too porous to properly support an ACV as the cushion air permeates the ice, not supporting the ACV at an adequate height.

The results presented herein may be useful in comparing the fuel costs of ACV icebreaking to conventional icebreaking.

5.6 Issues

In the ACV icebreaking fuel requirement analysis performed in this chapter, it was assumed that breakage in the wave action mode is due only to the dynamic ice wave effect discussed in Chapter 4. However, some researchers have attributed this breakage to a powerful stern wave produced in the water below the ice sheet by air from the ACV [5].

It is quite possible that both ice waves and water waves contribute to the overall effect. It is evident then that further research is needed at both the experimental and theoretical levels to verify the actual mechanism of fast mode ACV icebreaking. One possible experiment would be to test for the existence and pressure of water at the bottom of the ice sheet in the ice sheet stern wave. From the theoretical standpoint, a fluid mechanics analysis must be performed to determine exactly how a stern water wave with sufficient pressure to lift and break the ice cover could be formed by the passing ACV. Only with a solid understanding of the theoretical basis of ACV icebreaking can this system, which offers many potential advantages over conventional icebreaking, be used efficiently to improve Canadian transportation.

CHAPTER 6

FURTHER RESEARCH REQUIREMENTS

6.1 Introduction

In the preceding chapters the most important topics related to the transportation use of ice sheets were discussed, as was the current lack of an adequate understanding of many of the critical aspects of ice sheet behaviour. Research, both theoretical and experimental, is required to improve the safety and efficiency of ice sheet road transportation and to refine ACV icebreaking techniques.

This chapter will outline the theoretical and experimental research requirements of the following topics:

- ice properties and measurement techniques
- the static bearing capacity of ice sheets
- dynamic loads on ice sheets
- ACV icebreaking.

6.2 Ice Properties and Measurement Techniques

The safe use of ice sheet roads requires a clear understanding of the material behaviour of field ice and a knowledge of its mechanical properties. At this time uncertainties still exist respecting the viscoelastic behaviour of ice and

the most accurate means of determining the strength properties of a given ice sheet.

Theoretical research needed to clarify these uncertainties includes:

- Developing rheological models of different ice types.
- Developing from a theoretical standpoint the most accurate type of in-situ test for determining the strength properties of field ice relevant to its bearing capacity for vehicles.

Required experimental/empirical research on ice properties and measurement techniques includes:

- Improving the in-situ cantilever beam test to reduce disturbance to the natural state of field ice.
- Developing statistical correlations between ice strength values measured in standardized laboratory tests and the corresponding values measured in standardized in-situ tests.
- Further investigating ice creep behaviour in the field.

6.3 The Static Bearing Capacity of Ice Sheets

As discussed in Chapter 3, further research is needed in order to assure the safe use of ice covers to support static and quasi-static loads. The major problems are the ice sheet bearing capacity for loads of long duration and ice sheet cracking. The long-duration load bearing capacity problem has to date received little theoretical treatment and even less experimental examination. This must be clarified in order to improve safety in the increasing use of ice sheets for pulpwood landings, river construction platforms and off-shore drilling platforms.

Ice sheet cracking, a problem which has received almost no theoretical analysis, is a major concern of engineers operating ice sheet roads. This analysis, in addition to finding a better means of monitoring ice pressure build-up leading to cracks, is required in order to aid engineers in setting load restrictions and planning the ice road route.

Theoretical research needed includes:

- Developing a practical theory to determine the bearing capacity of ice sheets for static loads of long duration.
- Examining the bearing capacity of an ice sheet for a series of vehicles at different proximities for the purpose of determining optimally safe spacings. This could possibly be done by using the method of superposition with Wyman's Theory of the deformations of a floating elastic plate.
- Analyzing the effect of thermal stresses due to air temperature changes on ice sheet bearing capacity.
- Developing the capability to analyze where cracks and pressure ridges are most likely to occur on a given lake or river.

Required experimental/empirical research includes:

- Conducting additional field studies to obtain experimental curves showing the time variation of static breakthrough load due to the viscoelastic behaviour of the ice.
- Developing economical instrumentation to monitor ice pressure buildup along a road on ice.

6.4 Dynamic Loads on Ice Sheets

Chapter 4 has demonstrated the important discrepancies between Nevel's Theory of a moving load on a floating ice sheet and experimental observations of ice sheet dynamic response. Chief among these discrepancies is that Nevel's Theory

predicts an infinite deflection under a load travelling at critical velocity, whereas the limited experiments performed thus far have shown fairly low finite values (1.8 to 4.6) for the deflection amplification at critical speed. Although Nevel's Theory has generally predicted critical velocities to within 10% accuracy, it needs to be refined in order to obtain more realistic results respecting critical deflections.

In addition, theories must be developed to predict the dynamic effects in non-steady state problems commonly occurring in ice sheet road operation, such as an accelerating vehicle and vehicles travelling in close proximity at different speeds. The problem of the dynamic effects produced by a vehicle approaching shore is of major concern to engineers operating ice sheet roads, but it has not yet been treated theoretically nor formally observed in experiments. The dynamic wave phenomenon is important, representing both a safety hazard to land vehicles travelling over ice sheet roads and an operational aid to ACV icebreaking; an improved understanding of the phenomenon is imperative.

Theoretical research required in order to improve understanding of the dynamic wave phenomenon includes:

- Reformulating Nevel's analysis by using Timoshenko plate theory and generalizing the analysis to include initial horizontal stresses, fluid dynamic effects, viscoelasticity of the ice, anisotropic constitutive relations, ice temperature field, and non-uniform sheet thickness.
- Developing new theories to predict dynamic effects under the following common practical situations:
  1) An accelerating/decelerating vehicle.
  ii) Multiple vehicles at different proximities.
  iii) A vehicle approaching shore.

- Developing a theory to explain Eyre's finding that peak fracture activity occurs in an ice sheet when the load moves at about 85 per cent of critical velocity.

Experimental/empirical research required includes:

- Conducting further field experiments on the response of an ice sheet to dynamic loading, including accelerating/decelerating load movements and multiple loads, and examining the dynamic effects produced when approaching the shore.
- Conducting field experiments to verify Eyre's findings.
- Conducting dynamic load experiments on a laboratory floating ice sheet model under strict speed control to investigate the possibility that the critical deflection amplification factor is large but lies within a very narrow range of velocities.

6.5 ACV Icebreaking

As discussed in Chapter 5, ACV icebreaking offers many potential benefits to transportation in Canada, including the provision of icebreaking services to remote inland lakes inaccessible to existing icebreaking ships and the provision of icebreaking capability for conventional ships. At this time, however, knowledge of the actual mechanism of wave action (fast mode) ACV icebreaking is limited, and the relative importance of the established mechanisms involved in air cavity (slow mode) icebreaking is unknown. Further, field experimentation is required in order to identify the breakage mechanisms involved, and theoretical work is needed to predict and explain the mechanisms. The results of this research are required to develop design and operational guidelines for icebreaking ACV's.

Required theoretical research into ACV icebreaking includes:

- Developing a floating ice plate theory to analyze the several fracture mechanisms of slow mode icebreaking.
- Performing a fluid mechanics analysis to determine if a stern water wave with sufficient pressure to lift and fracture the ice cover could be formed by the passing ACV in fast mode icebreaking.

Experimental/empirical research requirements include:

- Further operational tests of both slow and fast mode ACV icebreaking. The fast mode tests should include measurements of the existence and pressure of water at the bottom of the ice sheet in the ice sheet stern wave to check the importance of the proposed water wave breakage mechanism.
- Developing equipment capable of measuring air/water pressure above and below the ice sheet, and the water depression beneath the ACV.

6.6 Concluding Remarks

The literature survey and discussions with engineers and active researchers conducted during this study have demonstrated a need to conduct further research on ice sheet behaviour as outlined above. Such research will facilitate safer, more efficient operations of ice sheet roads and ACV icebreaking in Canadian transportation.

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APPENDIX I

REMARKS ON D.E. NEVEL'S PAPER

"MOVING LOADS ON A FLOATING ICE SHEET"

APPENDIX I

REMARKS ON D.E. NEVEL'S PAPER "MOVING LOADS ON A FLOATING ICE SHEET"

(Adapted from Olesiak [19][1])

In Nevel's paper [17], the author derives Equation 31 which is, according to his own statement in the introduction, the same as Kheisin's (39)[2] derived in 1963. The Equation 31 gives the deflection at $x = r \cos \Theta$ and $y = r \sin \Theta$ (after Nevel's notation [17]) due to the force applied at the origin of the moving coordinate system:

$$W = \frac{P}{4\pi^2 \rho_1 g} \int_0^{\infty} \int_0^{2\pi} \frac{\exp[i\gamma r \cos(\Theta - \psi)]}{a - b \cos^2 \psi} \gamma d\gamma d\psi \qquad (1)$$

Transforming (1) to a cartesian coordinate system gives:

$$W = \frac{P}{4\pi^2 \rho_1 g} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\exp[i(ax + \beta y)]}{a - ba^2/\gamma^2} d\alpha d\beta \qquad (2)$$

where $a = 1 + \gamma^4 l^4$

$b = \lambda^2 \gamma \ (1 \coth \gamma H + \mu \gamma l^2)$

This is Equation 27 in Nevel's paper. To find a deflection at point x,y due to a distribution of forces over a certain region A, this equation must be re-written in such a way as to yield a value of deflection at point x,y due to a concentrated force P (ε, η) integrated over the region A. Therefore we have to start from the formula (see Olesiak [19] for definition of symbols):

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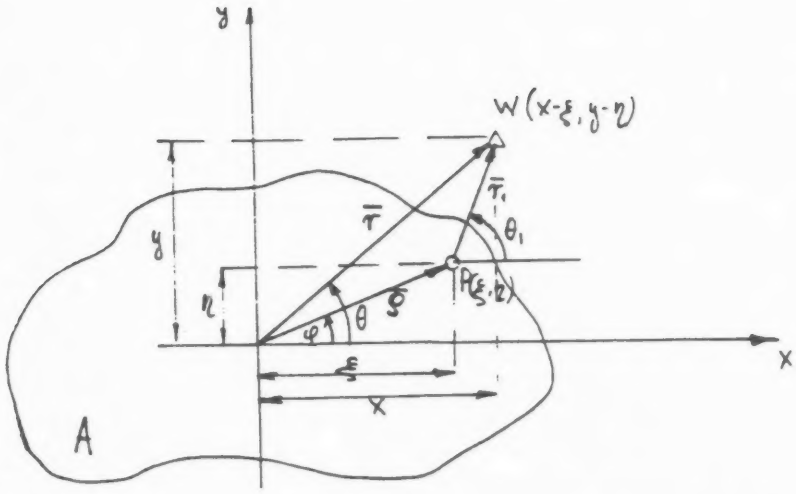

Fig. 1 Transformation to Polar Coordinates

$$W(X-\varepsilon,\ y-\eta) = \frac{P(\varepsilon,\ \eta)}{4\pi^2\rho_1 g} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\exp[i\{d(x-\varepsilon)+\beta(y-\eta)\}]}{a-\ ba^2/\gamma^2}\ d\alpha d\beta \qquad (3)$$

Now we change the cartesian coordinate system to the polar one as shown in Figure 1.

$$\begin{aligned} \alpha &= \gamma \cos\psi, & \beta &= \gamma \sin\psi, \\ x &= r \cos\Theta, & y &= r \sin\Theta, \\ \varepsilon &= \rho \cos\psi, & \eta &= \rho \sin\psi, \end{aligned}$$

where A denotes the area of the distributed load.

The total displacement at point x,y due to a distribution of pressure over the area A is given by the formula

$$w_{tot}(x,y) = \frac{1}{4\pi^2\rho_1 g} \int_A \int P(\varepsilon,\eta)\,d\varepsilon d\eta \cdot \int_{-\infty}^{+\infty} \int \frac{\exp\{i[d(x-\varepsilon)+\beta(y-\eta)]\}}{f(\alpha,\gamma)}\ d\alpha d\beta \qquad (4)$$

or in the polar coordinate system:

$$w_{tot}(r,\Theta) = \int_0^{\infty} \int_0^{2\pi} \frac{\exp[-ir\gamma\ \cos(\Theta-\psi)]}{a(\gamma)-b(\gamma)\cos^2\psi}\ d\gamma d\psi \cdot \int_A \int \rho P(\rho,\phi)\ \exp[-i\gamma\rho\ \cos(\phi-\psi)]\,d\rho d\phi \qquad (5)$$

If the load is represented by a circle such that $\rho < R$, $0 < \phi < 2\pi$ and the pressure is assumed constant, the following equation is obtained:

$$P \int_A \int = P \int_0^R \int_0^{2\pi} \rho \exp[-i\gamma\rho \cos(\phi-\psi)]d\rho d\phi = \frac{2\pi}{\gamma} J_1(\gamma R) \tag{6}$$

and consequently

$$W_{tot}(r,\Theta) = \frac{P}{2\pi\rho_1 g} \int_0^{\infty} \int_0^{2\pi} \frac{\exp[ir\gamma \cos(\Theta-\psi)]}{\alpha(\gamma)-b(\gamma) \cos^2\psi} J_1(\gamma R)d\gamma d\psi \tag{7}$$

This result does not agree with Equation 36 of Nevel; the integrals in Equation 36 are incorrect. This error carries throughout the remainder of Nevel's paper.

In Nevel, the discussion of the integrand in Equation 47 does not show precisely how the sign of the denominator depends on the magnitude of speed and the material constants. For shallow water this dependency was shown on page 7 of Olesiak [19].

Finally, Nevel's paper contains several misprints:

page 3, Equation (13):	$-u \frac{\partial \phi}{\partial x}$ should be $-u\frac{\partial w}{\partial x}$
page 4, Equation (17):	plus sign missing in the exponent (there is a comma in its place
page 4, Equation (21):	$z = w$ should be $z = H$
Equations (19)-(21):	should contain ordinary rather than partial derivatives.

APPENDIX II

THE CRITICAL VELOCITY OF A MOVING LOAD ON AN ICE BEAM SUBJECTED TO AXIAL COMPRESSIVE STRESS

APPENDIX II

THE CRITICAL VELOCITY OF A MOVING LOAD ON AN ICE BEAM SUBJECTED TO AXIAL COMPRESSIVE STRESS

(Taken from Long [15])

1. Introduction

When a floating ice sheet is subjected to changes in air temperature, thermal stresses are produced in the ice. These thermal stresses will be greatest when the temperature change is rapid such that viscous creep of the ice which acts to relieve the stresses cannot occur simultaneously with the temperature change. The temperature in the top portion of the sheet changes to the new air temperature while lower layers experience progressively smaller temperature changes and the bottom layer remains unchanged at about 0°C. A temperature drop induces contraction of the ice, but the ice sheet is at least partially constrained from compressing by its attachment to the lakeshore. Thus, tensile stresses are produced. Since ice is relatively weak in tension*, cracking is produced by significant temperature drops. This cracking may greatly reduce the load-carrying capacity of the ice sheet. Canadian operators of winter roads on ice are acutely aware of the dangerous conditions that result from rapid temperature drops and the usual practice is to close roads for a period of at least 24 hours following such changes. This allows the cracks to refreeze.

When cracks in the ice sheet due to temperature drops refreeze, however, the total amount of frozen water in the sheet increases.

* Representative values for the tensile and compressive strengths of freshwater ice are 1.5 MPa and 3.4 MPa respectively.

Then, if there is a subsequent large increase in air temperature, the sheet is restrained from its normal expansion by the shore attachments and compressive stress results. If the sheet is large and slender, buckling may occur causing the formation of pressure ridges (these are often observed on large lakes such as Lake Athabaska in northern Saskatchewan - Alberta). Otherwise, the sheet may be subjected to the constant compressive stress over a significant period of time until it is reduced by creep or changed again by changing temperature. The purpose of this chapter will be to examine the effects of this compressive stress on the critical velocity of loads on ice. Because an analysis for a floating ice plate subjected to horizontal stress is not available, the analysis presented herein will be for a floating ice beam. The results of both Euler-Bernoulli and Timoshenko beam analyses will be considered.

2. Assumptions

In examining the effect of compressive stress on the critical velocity of a load moving on a floating ice beam, the following simplifying assumptions will be made:

i) The compressive stress produced by a temperature increase is constant over the face of the beam.

ii) The beam is rigidly constrained at its ends, i.e. the ice is rigidly affixed to the shoreline.

The first assumption ignores the fact that there is a temperature gradient in the ice beam as discussed in 1. The exact shape of this gradient is variable, depending on the thickness and thermal characteristics

of the ice, so the assumption of a constant thermal stress distribution which implies that the resultant compressive force is axial is made to simplify the problem.

The second assumption ignores the possible freedom of the ends of the ice beam to move up or down the shoreline in response to changes in thermal stress. It is likely that the ends can move over the shore to partially relieve the stresses and that this allowed movement is a function of such factors as the shoreline slope and roughness. However, no studies appear to be available on this movement.

Further, the critical velocity on a buckled beam will not be considered. If the initial axial compressive stress exceeds the critical value given by equation (11), the ice beam will buckle, and then the equations used to find the critical velocities will give negative values. A further critical velocity analysis for the buckled beam is beyond the scope of this paper.

Assumptions (i) and (ii) allow the following expression to be written for the axial compressive stress in the ice beam as a function of the temperature increase:

$$N = -(E\alpha\Delta T) \qquad (1)$$

where: N = axial compressive stress in ice beam (N/m^2)
E = modulus of elasticity for ice = $4.9\ (10)^9\ N/m^2$
α = coefficient of thermal expansion for ice = 0.000051/°C
ΔT = temperature increase (°C)

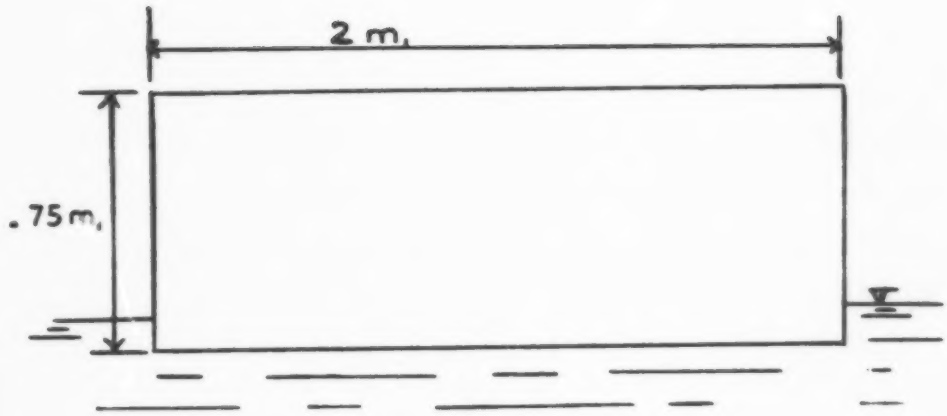

Figure 1: Cross-Section of Floating Ice Beam Used in Analysis

The study will consider the effects of a range of compressive stresses corresponding to temperature increases varying between 0 and 20 degrees Celsius.

3. Analysis and Results

3.1 Properties of the Ice Beam

The ice beam considered in the analysis is an infinite beam which rests on a continuous Winkler foundation of water such that the restoring force exerted on the beam by the foundation at a point is directly proportional to the transverse deflection of the beam at that point. That is:

$$R = kw \tag{2}$$

where: R = restoring force (N/m width)
k = foundation parameter (N/m^2)
w = deflection (m)

The beam has a section of width 2 m. and thickness .75 m., as shown in Fig. 1. The width is representative of that of a small vehicle and the thickness is representative of the ice thicknesses found on lakes in northern Canada in the winter. The section properties needed for analysis are:

$I = .070\ m^4$
$A = 1.50\ m^2$

where: I = moment of inertia
A = cross-sectional area

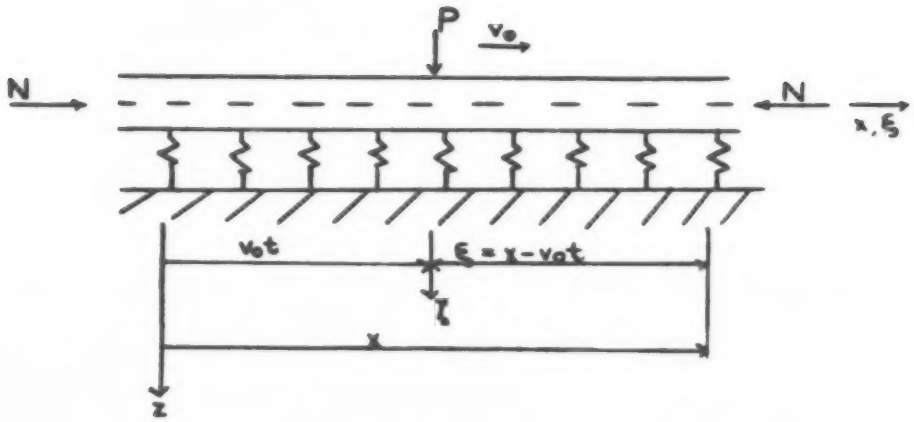

Figure 2: Illustration of Problem.

The mechanical properties of ice are highly variable. Representative values that are used in the calculations are:

$\rho = 917$ kg/m^3
$E = 4.9\ (10)^9$ N/m^2
$G = 1.8\ (10)^9$ N/m^2

where: ρ = mass density of ice
E = modulus of elasticity of ice
G = shear modulus of ice

3.2 Results Using Euler-Bernoulli Formulation

The problem of the continuously supported beam subjected to an axial force and a moving load was solved in 1972 by Kerr [30] using the Euler-Bernoulli Beam Theory. His derivation will be repeated here.

Consider an infinite beam resting on a Winkler foundation and subjected to a concentrated transverse load P moving at constant velocity v_0 and a constant axial force N, as shown in Figure 2. The analysis is based on the following differential equation of motion for an element of the beam:

$$EI\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} + kw = P\delta(x - v_0 t) \qquad (3)$$

where the new symbols are:
N = constant axial force
x = distance along beam
t = time
P = concentrated transverse load
v_0 = constant velocity of load

$\delta(x-v_0t)$ = Dirac's delta function such that

$\delta(x-v_0t) = 1$ if $x = v_0t$
$\delta(x-v_0t) = 0$ if $x \neq v_0t$

The foundation parameter K represents the load force required to cause a downward vertical unit displacement (1m.) of the beam across its full width. Since the foundation is water, its resisting force is simply the buoyant force equal to the weight of water displaced. For the unit displacement, therefore, the load force required per unit area is $9.81\ (10)^3\ N/m^2$ (per meter displacement). Since the beam is 2m. wide, the foundation parameter is:

$$K = 9.81\ (10)^3\ (2)\ \frac{N \times m}{m^2 \times m} = 19{,}620\ N/m^2$$

Because of the infinite extent of the beam and its constant properties, and also because of the assumption that v_0 = constant, it is reasonable to assume that after some time period the transient motions will become very small and the beam deflections will approach a steady state. Hence, the deflections of the beam will appear static from the vantage point of an observer travelling with the moving load, and the partial differential equation (3) may be transformed into an ordinary differential equation in the moving reference frame. With the new co-ordinate

$$\xi = x - v_0 t$$

(3) becomes

$$EI \frac{d^4 w}{d\xi^4} + (N + mV_0^2) \frac{d^2 w}{d\xi^2} + kw = P\delta\ (\xi) \qquad (5)$$

Because of the steady-state assumption, P moves at a constant altitude and thus does not accelerate in the z-direction. P in equation (3) therefore represents only the static intensity of the load.

Equation (3) can be solved by setting the right-hand side equal to zero and taking the concentrated load P into account later by matching conditions at $\xi = 0$.

For convenience, setting

$$4\alpha^2 = \frac{N+mv_0^2}{EI} \; ; \quad 4\beta^4 = \frac{k}{EI} \tag{6}$$

Equation (5) becomes

$$\frac{d^4w}{d\xi^4} + 4\alpha^2\frac{d^2w}{d\xi^2} + 4\beta^4 w = 0 \tag{7}$$

Assuming as a solution $w = Ae^{s\xi}$ and substituting it into equation (7) we obtain

$$s^4 + 4\alpha^2 s^2 + 4\beta^4 = 0 \tag{8}$$

The above equation is a quadratic in s^2. The four roots are

$$s_{1,2,3,4} = \pm\sqrt{-2[\alpha^2 \pm (\alpha^4 - \beta^4)^{1/2}]} \tag{9}$$

Noting that $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2)$, we must distinguish three cases:

$\alpha^2 \lessgtr \beta^2$ which corresponds to $v_0 \lessgtr \sqrt{\left(\frac{4kEI}{m^2}\right)^{1/2} - \frac{N}{m}}$ (10)

Note that the velocities v_0 are real only when

$$N < 2\sqrt{kEI} = N_{CR} \tag{11}$$

It is known that when $N > N_{CR}$ the beam has buckled and the differential equation (3) is not applicable.

In general, the roots in equation (9) are complex numbers of the form

$$s_{1,2,3,4} = \pm\lambda \pm j\omega \tag{12}$$

The corresponding solution for the region ahead of the load P is

$$w_2(\xi) = e^{+\lambda\xi}\left[A_1\cos(\omega\xi) + A_2\sin(\omega\xi)\right] + e^{-\lambda\xi}\left[A_3\cos(\omega\xi) + A_4\sin(\omega\xi)\right] \quad (\xi \geq 0) \tag{13}$$

And for the region behind the load

$$w_b(\xi) = e^{+\lambda\xi}\left[A_5\cos(\omega\xi) + A_6\sin(\omega\xi)\right] + e^{-\lambda\xi}\left[A_7\cos(\omega\xi) + A_8\sin(\omega\xi)\right] \qquad (14)$$
$$(\xi \le 0)$$

It is to be expected that as $\xi \to \infty$, $w_a = 0$, and as $\xi \to -\infty$, $w_b = 0$.

These conditions applied to (13) and (14) yield

$$A_1 = A_2 = 0;\ A_7 = A_8 = 0 \qquad (15)$$

The remaining four constants are determined from the four conditions at $\xi = 0$.

$$w_a(0) = w_b(0) = 0 \qquad (16)$$

$$\left.\frac{dw_a}{d\xi}\right|_0 = \left.\frac{dw_b}{d\xi}\right|_0 \qquad (17)$$

$$\left.\frac{d^2w_a}{d\xi^2}\right|_0 = \left.\frac{d^2w_b}{d\xi^2}\right|_0 \qquad (18)$$

$$\left(\frac{d^3w_a}{d\xi^3} - \frac{d^3w_b}{d\xi^3}\right) = \frac{P}{EI} \qquad (19)$$

The constants are determined as

$$A_4 = -A_6 = \frac{P}{4EI\omega(\lambda^2+\omega^2)} \qquad (20)$$

$$A_3 = A_5 = \frac{P}{4EI\lambda(\lambda^2+\omega^2)} \qquad (21)$$

Now consider the case when

$$v_0 = \sqrt{\left(\frac{4kEI}{m^2}\right)^{1/2} - \frac{N}{m}} \qquad (22)$$

TABLE 1: CRITICAL VELOCITY OF LOAD ON FLOATING ICE BEAM UNDER DIFFERENT AXIAL COMPRESSIVE STRESSES (EULER-BERNOULLI THEORY)

TEMPERATURE INCREASE (°C)	AXIAL STRESS (KN/m^2)	CRITICAL VELOCITY (Km/h)	CRITICAL VELOCITY (mph)	PERCENTAGE OF ZERO AXIAL STRESS CRITICAL VELOCITY
0	0	196.9	122.4	100
2	-500	177.8	110.5	90.3
4	-1000	157.0	97.6	79.7
6	-1500	132.5	82.3	67.2
8	-2000	102.2	63.5	51.9
10	-2500	58.7	36.5	29.8
12	-3000	<0 Buckling	<0 Buckling	-

This means that $\alpha^2 = \beta^2$. Hence, the roots of (8) are two complex double roots.

$$S_{1,2,3,4} = +/-\sqrt{2}\alpha j = +/-j\omega \qquad (23)$$

Thus $\lambda = 0$ as defined in (12), and the constants A_3 and A_5 become unbounded. The beam deflection therefore is unbounded along its entire length, and the velocity in (22) is the critical velocity.

Table 1 shows the critical velocities obtained for the Euler-Bernoulli ice beam from equation (22) when K, E, I, and m are replaced by the constant values which have been given and the compressive stress N is replaced by different values corresponding to different temperature increases. These results are discussed in section 4.

3.3 Results Using Timoshenko Formulation

In 1981, B. Prasad [31] analyzed the problem of the moving load on a beam supported on an elastic foundation using equations for the Timoshenko beam with an initial axial stress. These equations of motion had been previously derived by Sun in 1972. The hypothesis of Prasad was that the conventional Euler-Bernoulli beam equations are often inadequate for solving the moving load problem when initial axial stress is present.

The Timoshenko beam theory differs from the Euler-Bernoulli theory in that it takes into account the effect of shearing deflections and of rotatory inertia. Fig. 3 presents the notation and geometry of an unloaded element of a Timoshenko beam (not resting on an elastic foundation).

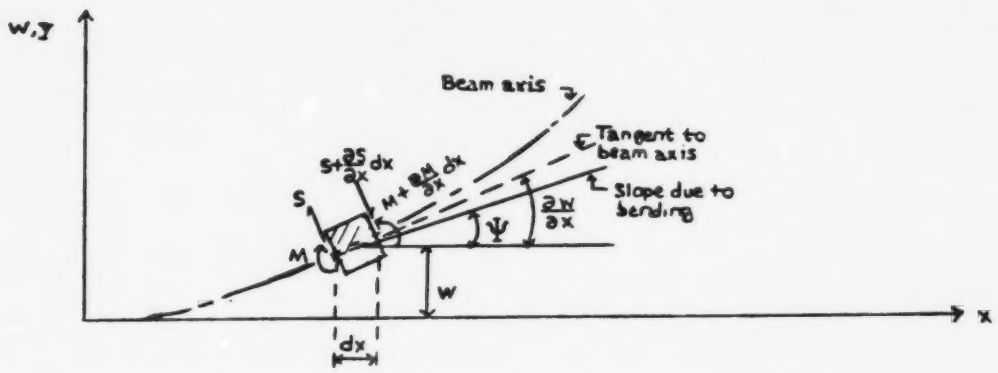

Figure 3 Notation and Geometry of an Element of a Timoshenko Beam.

It is seen that the total slope of the element is the sum of the slope due to bending ψ and the slope due to shear deformation $\frac{\partial w}{\partial x} - \psi$. The shear deflection coefficient κ is defined in the following elastic relation:

$$S = \kappa GA \left(\frac{\partial w}{\partial x} - \psi\right) \tag{24}$$

The value of κ is variable, depending on the geometry of the cross-section. For a typical railroad rail, its value is .822. For the rectangular ice beam section considered here. κ will be taken as unity.

The equations of motion for rotation and translation, respectively, of the beam element can be found from Fig. 3: for rotation

$$\Sigma M = I\ddot{\psi}$$

$$M + \frac{\partial M}{\partial x}dx - M - \left(S + S + \frac{\partial S}{\partial x}dx\right)\frac{dx}{2} = J dx \ddot{\psi}$$

where J = mass moment of inertia

or, expanding and then dividing through by dx

$$\frac{\partial M}{\partial x} - S = J\ddot{\psi} \tag{25}$$

for translation:

$$\Sigma F = ma \qquad \text{where F=force, m=mass, a=acceleration}$$

for forces in the y - direction

$$S - \left(S + \frac{\partial S}{\partial x}dx\right) = -\mu \ddot{w} dx$$

where μ = mass density per unit length

Expanding and then dividing through by dx

$$\frac{\partial S}{\partial x} = -\mu \ddot{w} \tag{26}$$

A further elastic relation between moment and deflection is:

$$M = -EI\frac{\partial \psi}{\partial x} \tag{27}$$

Eliminating M and S from equations (25) through (27) yields the following pair of coupled equations for free vibrations of the Timoshenko beam:

$$EI\frac{\partial^2 \psi}{\partial x^2} + \kappa GA\left(\frac{\partial w}{\partial x} - \psi\right) - J\ddot{\psi} = 0 \tag{28}$$

$$\mu\ddot{w} - \frac{\partial}{\partial x}\left[\kappa GA\left(\frac{\partial w}{\partial x} - \psi\right)\right] = 0 \tag{29}$$

The governing equations derived by Sun for the Timoshenko beam under an initial axial stress are:

$$(S + E)I\frac{\partial^2 \psi}{\partial x^2} + \kappa AG\left(\frac{\partial w}{\partial x} - \psi\right) = J\frac{\partial^2 \psi}{\partial t^2} \tag{30}$$

$$(S + \kappa G)A\frac{\partial^2 w}{\partial x^2} - \kappa AG\frac{\partial \psi}{\partial x} + p(x,t) = \mu\frac{\partial^2 w}{\partial t^2} \tag{31}$$

Prasad has extended (30) and (31) for the case of a beam being supported on a Winkler Foundation. This is done by replacing p(x,t) in equation (31) by $p(x,t) = F(x,t) - Q(x,t)$ (32)

where F (x,t) is the external load distribution = $F_o\delta(x-vt)$

, a point load travelling at velocity v (33)

and $Q(x,t) = kw + \beta_o\frac{\partial w}{\partial t}$ (34)

is the total restoring force of the foundation including viscous damping. Thus, equation (31) becomes:

$$(S+\kappa G)A\frac{\partial^2 w}{\partial x^2} - \kappa AG\frac{\partial \psi}{\partial x} - kw - \beta_o\frac{\partial w}{\partial t} = \mu\frac{\partial^2 w}{\partial t^2} - F_o\delta(x-vt) \tag{35}$$

Equations (30) and (35) are now the governing equations of motion for the Timoshenko beam under an axial force, supported on an elastic foundation, and subjected to a concentrated load of magnitude Fo moving at constant velocity v.

Using the concept of steady-state motion and with r = x - vt (position with respect to moving load), equations (30) and (35) may be reduced to two ordinary differential equations:

$$(S+E)I\frac{d^2\psi}{dr^2}+\kappa AG\left(\frac{dw}{dr}-\psi\right)=\rho Iv^2\frac{d^2\psi}{dr^2} \tag{36}$$

$$(S+\kappa G)A\frac{d^2w}{dr^2}-\kappa AG\frac{d\psi}{dr}-kw+\beta_0 v\frac{dw}{dr}=\rho Av^2\frac{d^2w}{dr^2}-F_0\delta(r) \tag{37}$$

Prasad has solved these equations by using the method of Fourier transforms. For the case of no damping ($\beta_0 = 0$), the following expression can be used to determine the critical velocity[1] of the load on the beam for different values of axial stress:

$$\Theta^4[1-\alpha]^2+2\alpha\Theta^2\left[2C_1+(1-\alpha)C_2-(1+\alpha)\frac{N'}{\alpha}\right]+(C_2\alpha+N')^2-4\alpha C_1C_2=0 \tag{38}$$

where: $\Theta=\dfrac{v_{cr}}{(G/\rho)^{1/2}}$

$\alpha=\dfrac{k(I/A)}{\kappa AG}$

$N'=\dfrac{N}{G}$

$C_1=\kappa+N'$

$C_2=\dfrac{E}{G}+N'$

v_{cr} = critical velocity

Equation (38) is a quadratic in θ^2, the other non-dimensional quantities being given by the section and material properties of the beam, the foundation modulus, and the assumed axial stress. In general, there can

[1] In theory, this critical velocity is the velocity at which the displacement becomes unbounded all along the beam.

TABLE 2: CRITICAL VELOCITY OF LOAD ON FLOATING ICE BEAM UNDER DIFFERENT AXIAL COMPRESSIVE STRESSES (TIMOSHENKO THEORY)

TEMPERATURE INCREASE (°C)	AXIAL STRESS (KN/m^2)	CRITICAL VELOCITY (Km/h)	CRITICAL VELOCITY (mph)	PERCENTAGE OF ZERO AXIAL STRESS CRITICAL VELOCITY
0	0	222.8	138.5	100
2	-500	206.6	128.4	92.7
4	-1000	188.6	117.2	84.6
6	-1500	168.8	104.9	75.7
8	-2000	146.5	91.1	65.7
10	-2500	119.9	74.5	53.8
12	-3000	85.7	53.3	38.5
14	-3500	16.6	10.3	7.4
16	-4000	<0 Buckling	<0 Buckling	-

be four roots to the equation, but only positive real roots are of practical significance. Once θ is determined, the critical velocity is given from the definition of θ as $v_{cr} = \theta \, (G/\varrho)^{1/2}$.

Table 2 shows the critical velocities at different axial compressive stresses obtained for the Timoshenko ice beam from equation (38). The constants in the equation were determined from the values for k, E, I, A and ϱ presented in sections 3.1 and 3.2 and an assumed value of $k = 1.0$. The calculation of critical velocities was performed by computer.

4. Discussion and Conclusions

The results shown in Tables 1 and 2 and plotted together in Fig. 4 indicate that the presence of initial axial compressive stress in an ice beam due to a temperature increase has a very significant effect on the critical velocity. For the particular representative beam used in the analysis, a rapid temperature increase of just 8^0C according to Euler-Bernoulli beam theory or 10^0C according to Timoshenko beam theory will reduce the critical velocity of the beam to about 50 percent of the original value with no initial compressive stress present.

There is, however, reason to believe that the effect of rapid temperature increase on the critical velocity has been overstated in the calculations. This overstatement is a result of the two assumptions made in section 2. The first assumption, that the compressive stress caused by a temperature increase is constant over the face of the ice beam, overstates the resultant compressive force. In actuality, there is a temperature change gradient in the ice layer with zero temperature change at the bottom, and

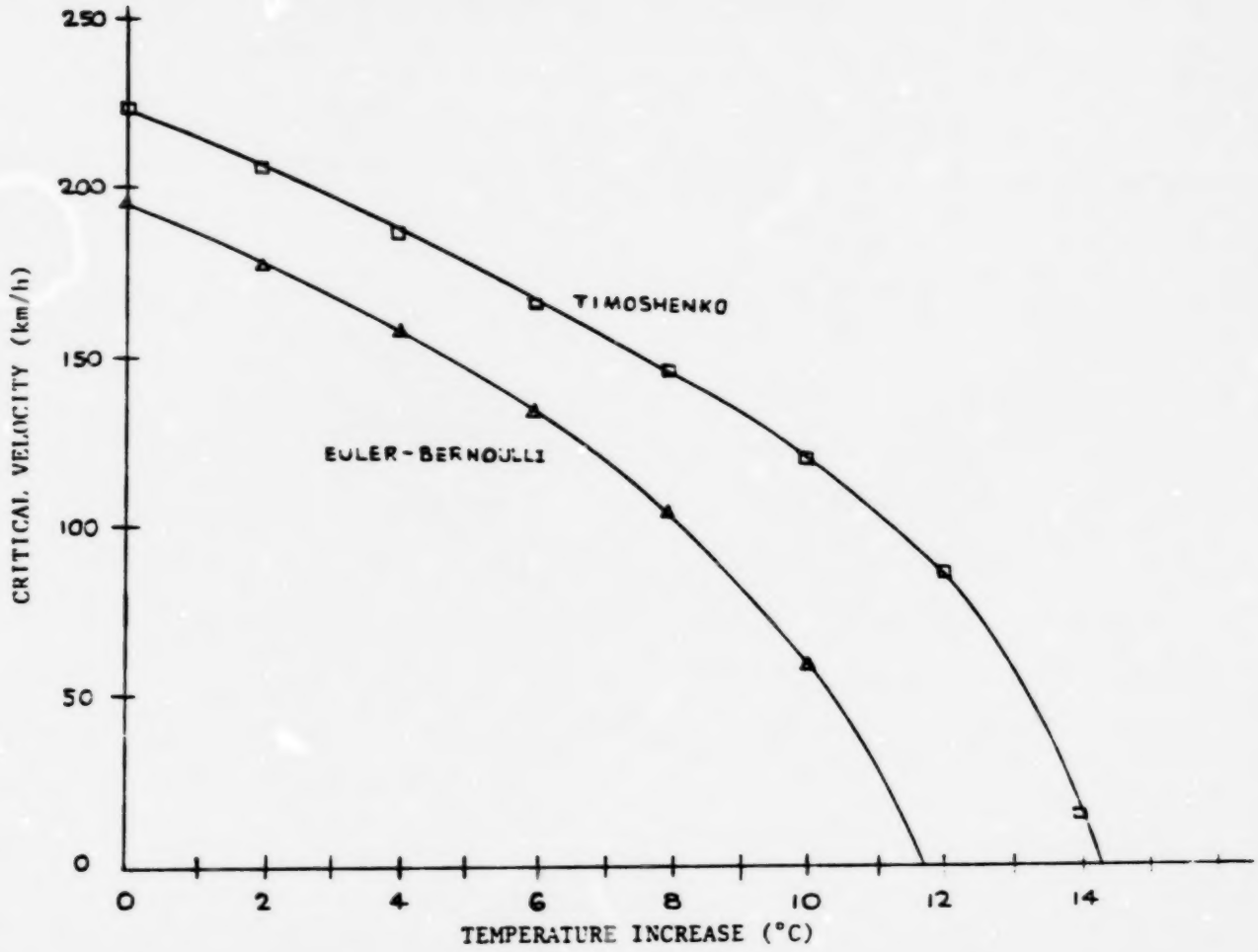

Figure 4 Critical Velocity of Load on Floating Ice Beam as a Function of Temperature Increase for Euler-Bernoulli and Timoshenko Theories.

hence the resultant force in the field may be much less than $-E\alpha\Delta TA$. As well, the temperature change gradient means that in the field the compressive stress is not axial and hence the analysis performed here has neglected an additional moment term in the differential equation of motion which may or may not be significant. The second assumption, that the ice is rigidly affixed to the shoreline, also causes an overstatement of the effect of temperature on critical velocity, because it is very likely that in the field the ends of the ice beam are allowed to move over the shoreline to partially relieve the thermal stress. Nevertheless, it would still appear that rapid temperature changes in the range of those commonly observed in Canada can have a very significant reducing effect on the critical velocity of a floating ice beam.

As seen from Fig. 4, analysis of the problem using the Timoshenko beam theory produces somewhat higher critical velocities than those obtained using Euler-Bernoulli theory and the critical velocity reduces at slower rates with temperature change for the Timoshenko case. These observed differences suggest that a revision of Nevel's analysis of a moving load on a floating ice plate [17] using Timoshenko theory for the plate would be worthwhile from the standpoint of possibly giving significantly different results.

The values of critical velocity at zero initial axial stress obtained in the analysis of the ice beam are considerably higher than those predicted by Nevel's theory (and observed in the field) for ice plates.[2] This is

[2]For a plate thickness of .75m = 30 in., Nevel's graph gives a maximum critical velocity = 32 mph.

to be expected intuitively from equation (22) which shows that the critical velocity decreases as the I/A^2 ratio decreases. Of course, the analysis of the moving load problem on a floating ice plate, the results of which are much more practical to operators of winter roads on ice, is considerably more complicated than that of the floating beam. However, the results of the study in this chapter strongly indicate the importance of performing an analysis of the response to a moving load of a floating ice plate subjected to initial compressive stress.

POOR

QUALITY

ORIGINAL